中國文化大學 101 學年度暑假轉學招生考試

系組：應用數學系三年級　　日期節次：7 月 24 日第 1 節 09:00-10:20

科目： 線性代數　　U-31

試題中，A，B代表矩陣. A^T 代表A的轉置矩陣. A^{-1}代表A的反矩陣.

1~4 題爲簡答題，每題 5 分，只須回答 True 或 False：

1. If A is n by n, then A and A^T have the same determinant.
行列式

2. If A, B, and C are matrices, then $(AB)C = A(BC)$.

3. If A and B are invertible, then $A+B$ is invertible.

4. If A and B are symmetric, then $ABAB$ is symmetric

5~12 題爲計算證明題，每題 10 分，必須寫清楚每一題的詳細過程：

5. Find a 2 by 2 matrix A such that $A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

6. Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. Find A^{-1}.

7. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$. Find the rank of A^T.

8. Find a basis for the nullspace of $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

9. Prove or disprove: The columns of a matrix are a basis for the column space.

10. Prove that if A and B are orthogonal matrices, then AB is also an orthogonal matrix.

11. Let $A = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$. Find the eigenvalues and eigenvectors of A.

12. Prove: If A is positive definite, then A^{-1} is also positive definite.