行政院國家科學委員會專題研究計畫成果報告

探針量測三維表面之取養間隔研究

The Effect of the Sampling Interval on the Stylus Measuring Three-Dimensional Rough Surface

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一、中文摘要

本研究使用頻譜分析及電腦模擬，研究探針量測三維表面時之路徑，找出最小取樣間隔的選擇依據。

關鍵字：探針、表面量測

Abstract

Spectral analysis and computer simulation are adopted to investigate the effect of sampling interval on tracing of three-dimensional isotropic rough surfaces with a spherical tipped stylus. By using the concept of critical wave number, the criteria for choosing sampling interval is established.

Keywords: stylus, surface measurement

1. Introduction

The effect of the stylus tip size on measuring errors of statistical parameters of rough profiles was investigated in several places [1-5]. But these researches are focused on the profile measurement.

As for the surface measurement, few researches were done [6, 7]. These researches are limited in special cases.

The object of the present study is to investigate the effect of the tip curvature of the stylus on measuring isotropic surface roughness. Wu's [4] approach is used, and is extended to three dimensions. Numerical simulation is employed to find the trend of the spectral density of measured surfaces. Finally, the criteria for selection of sampling interval are established.

2 Mathematical Model of Stylus Locus

The scheme for finding the locus of the stylus tip is similar to that of Mendeleyev [2].

The mathematical model of the surface tracing was developed with the following assumptions:

(1) The stylus tip has a spherical shape with a radius r.

(2) The lower hemisphere of the stylus tip contacts the surface.

(3) The traced surface is drawn by the lowest point of the stylus tip.

(4) The stylus tip and the rough surface have no plastic or elastic deformations.

3. Spectral Analyses

Spectral analyses are used in this paper. The profile spectral density can be found from its surface spectral density. According to Longuet-Higgins [8], in the μ_x direction, the profile spectral density (PSD) is

$$S^p(\mu_x) = \int_{-\infty}^{\infty} S(\mu_x, \mu_y) d\mu_y \qquad (1)$$

4. Surface Generation Method

Mulvaney et. al. [9] investigated 144 surfaces. They found that their profile PSD's are

$$S^p(\mu) = \frac{b\pm^2}{®(1+b^2\mu^2)} \qquad (2)$$

Their ACF is [10]

$$R(x) = \pm^2 \exp(-\frac{x}{b})$$

For isotropic surfaces, the surface auto-correlation function (ACF) is

$$R(x,y) = \pm^2 \exp(-\frac{\sqrt{x^2+y^2}}{b})$$

The surface PSD is

$$S^s(\mu_x,\mu_y) = \frac{b^2\pm^2}{2®[b^2(\mu_x^2+\mu_y^2)+1]^{3/2}}$$

By using Wu's [11] method, the surface can be generated. The simulated surface can be generated by

$$z_{p,q} = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1}\sqrt{\tilde{S}_{k,l}}\exp\left(i2\pi\left[\phi_{k,l}+\frac{kp}{M}+\frac{lq}{N}\right]\right) \quad (3)$$

$$p = 0,1,2,\cdots,(M-1)$$

$$q = 0,1,2,\cdots,(N-1)$$

where $\phi_{k,l}$ is a set of independent random phase angles, which makes $z_{p,q}$ to be real.

$\tilde{S}_{k,l}$ is used to generate the surfaces. For a two-dimensional surface, $\tilde{S}_{k,l}$ could be obtained from its real PSD by

$$\tilde{S}_{M-k,N-l} = \tilde{S}_{k,N-l} = \tilde{S}_{M-k,l} = \tilde{S}_{k,l} = S_{k,l}$$

where $S_{k,l} = S^s(\frac{k\pi}{\Delta x},\frac{l\pi}{\Delta y})\frac{\pi}{(M\Delta x)}\frac{\pi}{(N\Delta y)}$

5. Spectral Density of Measured Surface

Church and Takacs [3] made a conjecture that the spectrum of the measured profile is the same as that of the original profile up to a critical wave number, μ_c, above which it falls off as μ^{-4}.

$$S^p(\mu) = \begin{cases} \dfrac{b\sigma^2}{\pi(1+b^2\mu^2)} & \text{for } \mu \le \mu_c \\ S(\mu_c)(\dfrac{\mu_c}{\mu})^4 & \text{for } \mu \ge \mu_c \end{cases}$$

As for the surface measurement, it should be similar. Since the PSD of isotropic surface is symmetric about the origin. The PSD for an isotropic rough surface can be written in polar form as

$$S^s(\mu_r,\mu_\theta) = S^s(\mu_r) = \frac{b^2\sigma^2}{2\pi(b^2\mu_r^2+1)^{3/2}}$$

As for the measured surface, the PSD could be in the form of

$$S^s(\mu_r) = \begin{cases} \dfrac{b^2\sigma^2}{2\pi(1+b^2\mu_r^2)^{3/2}} & \text{for } \mu_r \le \mu_c \\ \dfrac{b^2\sigma^2}{2\pi(1+b^2\mu_c^2)^{3/2}}(\dfrac{\mu_c}{\mu_r})^p & \text{for } \mu_r \ge \mu_c \end{cases}$$

p is the exponent, which is not necessarily 4. μ_c is the critical wave number. It is related to the curvature of tip of stylus.

6. Simulation and Spectral Analysis

6.1 Simulation of Rough Surfaces

We study five different surfaces measured by three different sizes of styli.

$b = 5, 10, 20, 50, 100\ \mu\text{m}$

$r = 10, 20, 50\ \mu\text{m}$

with sampling interval $1\ \mu\text{m}$.

6.2 The locus

Figure 1 shows part of profile and locus of stylus for $b = 5\ \mu\text{m}$, $r = 50\ \mu\text{m}$. This is a typical case for a large stylus measuring a rough surface. In this figure, dotted line is found by treating the stylus as a sphere, and dashed line is found by treating the stylus as a circle. It is found that when rough surfaces are measured by larger stylus, the measured profiles are smoother.

6.3 Surface Spectral Densities of Measured Surfaces

Surface PSD's of measured surfaces are analyzed. Welch's averaged periodogram [12] is used to find the PSD's of the surfaces. In 2-D FFT, circular Hanning window is used to avoid side leakage [13, 14].

Figure 2a shows the surface PSD for the surface measured by a spherical stylus tip. Figure 2b shows the contours of the surface PSD. They are symmetric about the origin. Thus, we can plot the surface PSD in the radial direction.

Figure 3 shows the surface PSD in the radial direction in log scale. The dash-dot lines are based on equation (2) and the solid lines are the best-fit lines. We find that, below the critical wave number, the surface PSD for the measured surface is approximately equal to that of the original surface; and above that, the surface PSD for the measured surface is smaller. While above critical wave number, the surface PSD for the measured surface follows a power law. The exponent is between -4 and -5. That is, for the measured surface, the surface PSD can be written as

$$S^s(\mu_r) = \begin{cases} \dfrac{b^2\sigma^2}{2\pi(b^2\mu_r^2+1)^{3/2}} & \text{for } \mu_c \ge \mu_r \\ \dfrac{b^2\sigma^2\mu_c^p}{2\pi(b^2\mu_c^2+1)^{3/2}}\dfrac{1}{\mu_r^p} & \text{for } \mu_c \le \mu_r \end{cases} \quad (4)$$

The critical wave number μ_c is shown in table 1. The exponent p is in table 2.

6.4 Critical Wave Number and Minimum Useful Sampling Interval

From the previous deduction, we know that, there is a least sampling interval that should be used (related to the stylus size) since the additional effort in using a smaller sampling interval will simply generate meaningless data. The smallest sampling interval that should be used can be decided from the critical wave number as

$$s = \frac{®}{\mu_c}$$

The minimum useful intervals for all the cases we investigate are shown in Table 3.

6.6 Estimating the rms Error by Measured Spectral Density

Since the surface PSD can be predicted, the rms roughness can be predicted, too. Theoretically, rms roughness could be obtained by

$$\pm^2 = \int_0^{2®} d\mu\text{–} \int_0^{\infty} S(\mu_r, \mu\text{–})\mu_r d\mu_r$$

Using equation (5), the rms roughness can be obtained as

$$\pm^2_{measured} = \pm^2 \left(1 - \frac{1}{(b^2\mu_c^2 + 1)^{1/2}} + \frac{b^2\mu_c^2}{(p-2)(b^2\mu_c^2 + 1)^{3/2}}\right)$$

The fractional measuring error on rms roughness is

$$\sim = \frac{\pm - \pm_{measured}}{\pm} \qquad (5)$$

which is a function of $b\mu_c$.

Figure 4 shows the rms roughness obtained from equation (5) and that directly computed from measured and original surfaces. It is found that the trend is the same with only few errors.

6.7. Non-Dimensionalization

The measured surfaces are decided by: r, b, $\pm$. These three parameters can be reduced into two by non-dimensionalization.

$$b' = \frac{b}{\pm} \qquad r' = \frac{r}{\pm}$$

The non-dimensional PSD becomes

$$S^{s}{}'(w_x, w_y) = \frac{b'^2}{2®[b'^2 (w_x^2 + w_y^2) + 1]^{3/2}}$$

or, in polar form,

$$S^{s}{}'(w_r) = \frac{b'^2}{2®(b'^2 w_r^2 + 1)^{3/2}}$$

Therefore, the result in this paper can be extended to all isotropic cases.

6.8 Application

If we use a stylus with radius $r = 20$ «m and measure a surface with rms roughness $\pm = 2$ «m and correlation distance $b = 40$ «m . Then, $b = 20$ «m , $r = 10$ «m . So, the minimum useful sampling interval is $s = 13.8$ «m .

7. Conclusion

This paper uses spectral analyses and computer simulation to investigate the effect of stylus tip radius on measuring isotropic surfaces. For the measured surfaces, the critical wave number and the power at high wave number are found. Therefore, the criteria for selection of sampling interval are established.

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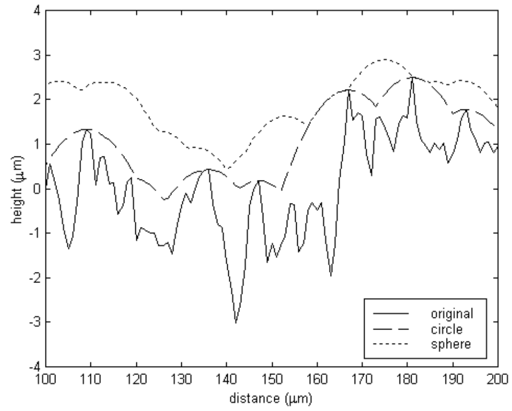

Figure 1. $b = 5\ \mu m$, $r = 50\ \mu m$, true profile and locus of stylus tip.

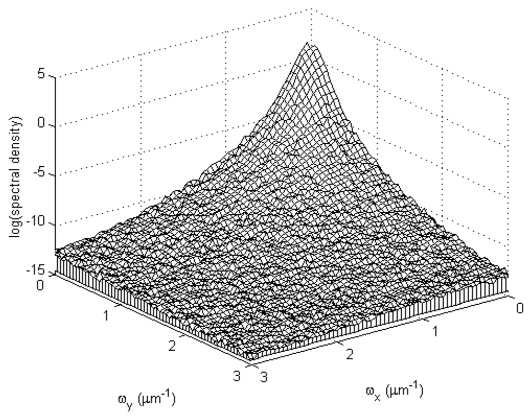

Figure 2a. Surface spectral density for $b = 5\ \mu m$, $r = 50\ \mu m$.

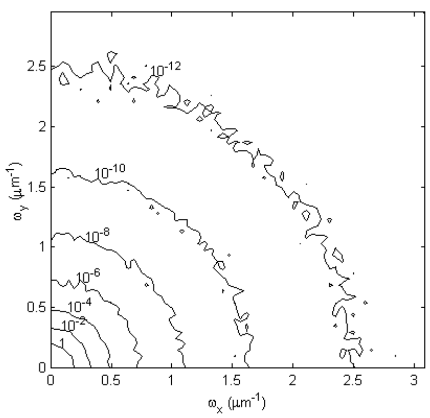

Figure 2b. Contours of surface spectral density for $b = 5\ \mu m$, $r = 50\ \mu m$.

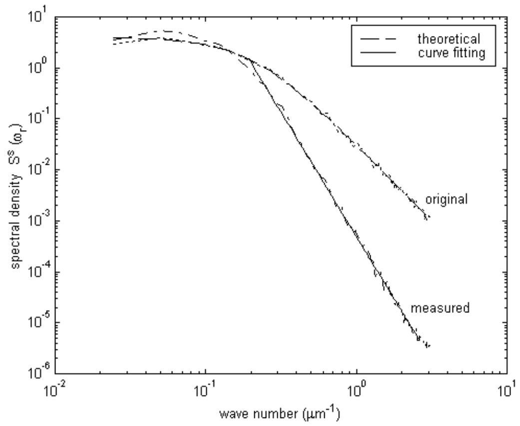

Figure 3. $S^s(\mu_r)$ for $b = 5\ \mu m$, $r = 50\ \mu m$.

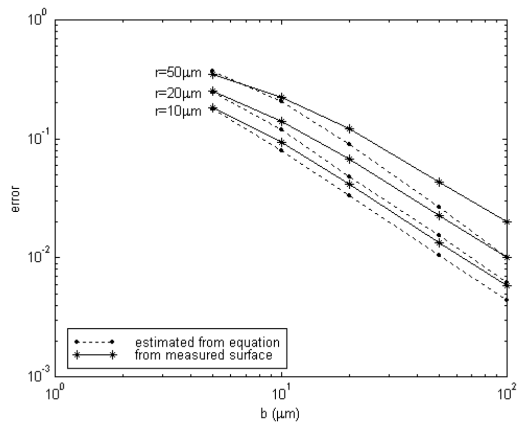

Figure 4. fractional error for rms height.

Table 1. Critical wave number (μm^{-1})

r \ b	5	10	20	50	100
10	0.3855	0.3992	0.4568	0.5358	0.5636
20	0.2857	0.2848	0.3417	0.4007	0.4802
50	0.1887	0.1726	0.1898	0.2381	0.3203

Table 2. Exponent at high wave Number

r \ b	5	10	20	50	100
10	4.64	4.53	4.45	4.27	3.99
20	4.79	4.70	4.75	4.54	4.46
50	4.83	4.79	4.81	4.71	4.78

Table 3. Minimum Reliable Sampling Interval (μm)

r \ b	5	10	20	50	100
10	8.149	7.870	6.877	5.863	5.574
20	10.99	11.03	9.193	7.840	6.542
50	16.65	18.20	16.56	13.19	9.809

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