探針量測表面粗糙度之取樣間隔之研究

Effect of Sampling Interval on the Measurement of Rough Surfaces with Stylus Instrument

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中文摘要

本研究以電腦模擬與頻譜分析來研究探針量測時取樣間隔的影響。結果發現取樣間隔會影響頻譜與量得之粗糙度。最適取養間隔與碎形表面之特性有關。

關鍵字：探針、取樣間隔

Abstract

Spectral analysis and computer simulation are adopted to investigate the effect of sampling interval on measuring rough surfaces. It is shown that spectral density can be affected by sampling interval. The optimal sampling interval strongly depends on the fractal dimension D and the non-scale parameter G.

Keywords: stylus, sampling interval

1. Introduction

The effect of the stylus tip size on measuring errors of statistical parameters of rough surfaces was investigated in several places [1-2]. Another approach was taken by Church and Takacs [3]. They studied the spectral density of measured surface. Wu [4] also used the spectral analysis and found a "critical sampling interval".

The object of the present study is to investigate the effect of the sampling interval on measuring fractal surface roughness. In this paper, spectral analysis is adopted. Finally, formulas are developed to estimate critical sampling interval.

2 Mathematical Model of Stylus Locus

The scheme for finding the locus of stylus tip is similar to that of Wu [4].

In developing a computer model, a surface is approximated as a discrete series of points. The stylus locus was assumed that:

(1) The radius of the stylus tip is r.

(2) The lower semicircle of the stylus tip contacts the surface.

(3) The traced profile is drawn by the center of the circle of the stylus tip.

(4) No plastic or elastic deformations.

The contact point has coordinates $X(J)$, $Y(J)$ and the stylus center has coordinates $X(I)$, $Z(I)$. The index J of the point of contact is determined by a search over the discrete points spanned by the stylus tip for a maximum of the function

$$h(J)+Y(J)=\underset{k}{Max}[h(k)+Y(k)]=Z(I)$$

where $h(k)=[r^2-(k-I)^2(\Delta X)^2]^{1/2}$

The range of indices k's are searched extends from $I-[r/\Delta X]$ to $I+[r/\Delta X]$.

These equations are used to simulate the trace of stylus.

3. Profile Generation

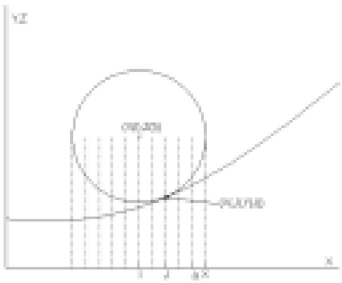

Figure 1. Stylus center and contact point coordinates

In this paper, we focus on the spectral analysis. Since the spectral density of rough surface at high wave number always has fractal properties. Therefore, fractal profiles are generated. For a fractal profile, its spectral density is of the form

$$P(\omega) = \frac{G^{2D-2}}{\omega^{5-2D}} \qquad (1)$$

G is a constant. Its dimension is ($length$).
D is the dimension, where $1 < D < 2$.

The Fourier filtering method [5] is used to generate fractal profiles. First of all, the discrete spectral density is made by

$P_k = P(\omega_k)\Delta\omega \quad$ for $k = 0, 1, 2, \cdots N/2$

Therefore, the Fourier Transform of generated profile is

$$Z_0 = \sqrt{P_0}$$
$$Z_k = \sqrt{P_k}(\cos\phi_k + i\sin\phi_k) \quad k = 1, 2\cdots, (N/2-1)$$
$$Z_{N-k} = Z_k \qquad \text{for } N > k > N/2$$

where ϕ_k is a set of random phase angles uniformly distributed between 0 and 2π.

The profile $\{z_l\}$ can be obtained by the following equation.

$$z_l = \sum_{k=0}^{N-1} Z_k \exp(i\frac{2\pi kl}{N}) \qquad l = 0, 1, 2\cdots, (N-1)$$

4. Spectral Density of Measured Profile

In 1991, Church and Takacs [5] made a conjecture that a measured profile should have the following spectral density.

$$P(\omega)_{measured} = \begin{cases} P(\omega) & \text{for } \omega \le \omega_c \\ P(\omega_c)(\frac{\omega_c}{\omega})^4 & \text{for } \omega \ge \omega_c \end{cases}$$

For a fractal profile, the spectral density of a measured profile will be

$$P(\omega) = \begin{cases} \frac{G^{2D-2}}{\omega^{5-2D}} & \text{for } \omega \le \omega_c \\ \frac{G^{2D-2}}{\omega_c^{5-2D}}\frac{1}{\omega^4} & \text{for } \omega \ge \omega_c \end{cases}$$

Church and Tacas thought that the critical wave number is implicit defined by the equation $\quad C\kappa^2_{stylus} = \sigma^2_\kappa \qquad (2)$

5. Simulation and Spectral Analysis

5.1 Non-Dimensional Analysis and Profile Generation

D and G are non-dimensionalized by

$$D \qquad g = \frac{G}{r}$$

Also, we define a non-dimensional wave number and non-dimensional spectral density as following.

$$w = r\omega \qquad H = g^{2D-2}$$
$$T(w) = \frac{P(\omega)}{r^3} = \frac{g^{2D-2}}{w^{5-2D}} = \frac{H}{w^{5-2D}}$$

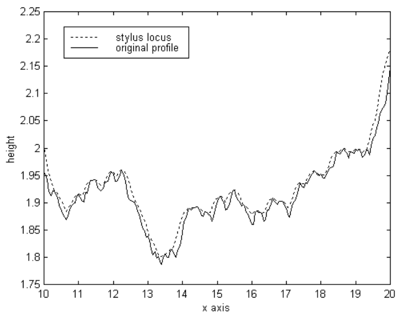

Figure 2. $D = 1.3$, $H = 10^{-3}$, profile and locus of stylus tip

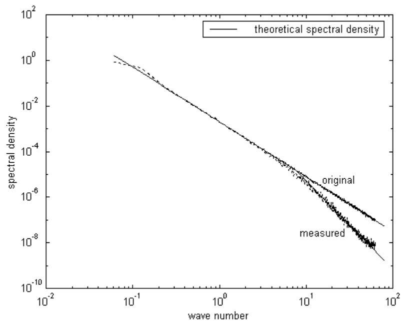

Figure 3. $T(w)$ for $D=1.3$, $H=10^{-3}$.

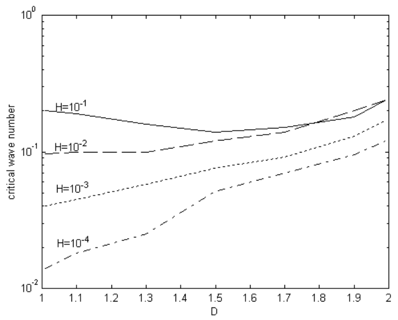

Figure 4. C

5.2 Result

We study four different H's with seven different fractal dimensions.

$D=1.01, 1.1, 1.3, 1.5, 1.7, 1.9, 1.99$

$H=10^{-1}, 10^{-2}, 10^{-3},\ 10^{-4}$

Totally, 35 cases are investigated. For each case, ten profiles are generated. For each profile, 32684 points are generated. The sampling interval is 0.05.

Figure 2 shows part of locus and profile and figure 3 shows the spectral densities for original and measured profiles of $D=1.3$, $H=10^{-3}$. If is found that, the spectral density of measured profile is the same as that of the original profile up to a critical wave number, w_c, above which it follows a power law $-D_h$. D_h is between 3 and 4. We use the least square to fit the data in the log-scale figure. We can find the critical wave number w_c and the power D_h for each profile.

We calculate the average C, w_c and D_h for each case. The C's are shown in figure 4. Figure 5 shows the average power D_h. Figure 6 shows the critical wave number w_c.

In figure 4, we find that C is not 0.1 as Wu claimed [6]. Therefore, the critical wave number can not be obtained from equation (2).

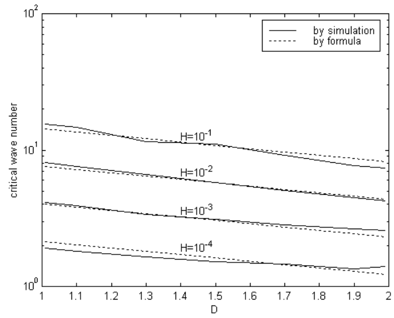

Figure 5. ω_c

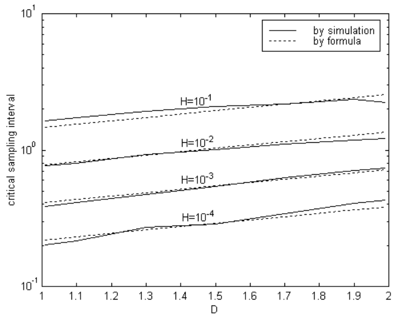

Figure 6. Critical sampling interval.

5.3 Critical Wave Number

Using regression, we can find an empirical formula for critical wave number.

$$\log w_c = -0.275\log H - 0.564D + 0.695$$

The critical wave number obtained from simulation and the one obtained from the above equation is shown in Figure 5.

5.4 Reliable Sampling Interval

The reliable sampling interval can be

decided by critical sampling interval as follow $s = \frac{\pi}{w_c}$

The critical sampling interval for all the cases we investigate are shown in Figure 6.

5.5 Estimating the Error of Variance of Height by Measured Spectral Density

The spectral density is approximated by

$$T(w) = \begin{cases} \dfrac{g^{2D-2}}{w^{5-2D}} & \text{for } w \le w_c \\ \dfrac{g^{2D-2}}{w_c^{5-2D}} \dfrac{1}{w^4} & \text{for } w \ge w_c \end{cases}$$

The variance of height is

$$\sigma^2 = 2\int_{w_s}^{w_h} T(w)\,dw$$

Thus, fractional measuring error on variance of height is

$$\alpha = \frac{\sigma^2 - \sigma^2_{measured}}{\sigma^2} = 1 - \frac{\dfrac{4-2D}{3w_c^{5-2D}}\left(\dfrac{1}{w_c^3} - \dfrac{1}{w_h^3}\right)}{\left(\dfrac{1}{w_s^{4-2D}} - \dfrac{1}{w_h^{4-2D}}\right)}$$

Examples are shown in figure 7. It is found that this equation can estimate the error of variance of height correctly.

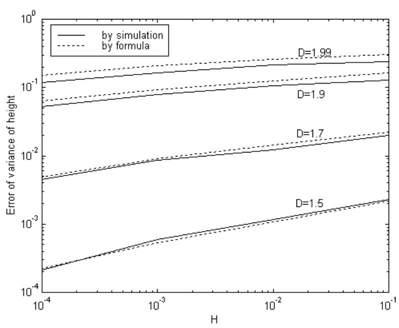

Figure 7. Error of variance of height

6. Conclusion

This paper use spectral analysis and computer simulation to investigate the effect of sampling interval on measuring fractal profiles. It is found that if the sampling interval is too large, the data about the surface may be too few. If the sampling interval is too small, the spectral density may be distorted. The reliable sampling interval strongly depends on H and D. The formula for critical sampling interval is obtained. From the critical sampling interval, the spectral density of measured profile is developed. Also, the error of rms height can be easily estimated.

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