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自旋預扭提摩新格樑受周期性變動軸向壓力作用下之參數不穩定分析
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PARAMETRIC INSTABILITY OF SPINNING PRETWISTED TIMOSHENKO BEAMS UNDER COMPRESSIVE AXIAL PULSATING LOADS

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PARAMETRIC INSTABILITY OF SPINNING PRETWISTED TIMOSHENKO BEAMS UNDER COMPRESSIVE AXIAL PULSATING LOADS

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ABSTRACT

Using the Timoshenko beam theory and applying Hamilton's principle, the present study establishes bending vibrations of a spinning pretwisted beam subjected to a harmonically time-dependent compressive axial force. Equations of motion of the twisted beam are derived in the spinning twist coordinate frame. The finite element method is employed to discretize the equations of motion into time-dependent ordinary differential equations with gyroscopic terms. A set of second-order ordinary differential equations with periodic coefficients of Mathieu-Hill type is formed to investigate the effect of twist angle, spinning speed, static component of axial force, length-to-width ratio and restraint condition on the parametric instability of the beam. It is believed that the present model is valuable for the parametric studies to understand better the various dynamic aspects of the twisted beam affecting its dynamic stability behavior.

Keywords: pretwist, Timoshenko beam, gyroscopic, parametric instability

1. INTRODUCTION

Vibration and stability analyses of the pre-twisted structures, such as turbine blades, propeller blades and drill bits, have received many researchers' attention in different engineering fields for past decades. In general, the dynamic behaviors of the pre-twisted structures have been analyzed based on Euler beam theory or Timoshenko beam theory. In addition to the geometric aspects, the effects of the rotational speed and axial load on the vibration and stability of twisted beams have also been studied by several scientists and engineers. A general review of the dynamic aspects of pre-twisted beams can be found in review paper by Leissa [1] and Rosen [2].

Based on Euler-Bernoulli beam theory, the effect of the twist angle on the natural frequencies and mode shapes of the cantilever turbine blade has been studied by Carnegie [3], Dawson and Carnegie [4], and Sabuncu [5]. By treating the blade as a pre-twisted Timoshenko beam, the vibration equations of motion of the pre-twisted blade were developed using different techniques by Carnegie [6], Dawson et al. [7], Gupta and Rao [8], Abbas [9], Subrahmanyam et al. [10], Lin et al. [11], Banerjee [12], and Yardimoglu and Yildirim [13] to study the effects of geometric aspects, rotary inertia and shear deformation on the lateral frequencies of the blade. Above studies were concentrated mainly on the free vibration characteristics of nonrotating pre-twisted beam structures.

The dynamic behaviors of rotating beams about a longitudinal or transverse axis have been investigated extensively related to the vibration of shafts, turbine blades and drill bits. A rotating twisted Timoshenko beam which rotates about an axis normal to the longitudinal axis of the beam was investigated by Fu [14], Rao and Carnegie [15], Subrahmanyam and Kaza [16], Rao and Gupta [17], and Sabuncu and Evran [18]. The lateral vibration equations of motion of the axially spinning pre-twisted beam have been established by Tekinalp and Ulsoy [19,20], and Lee [21-23] to study the effects of the spinning speed, the pre-twisted angle and aspect ratio of the beam on the natural frequencies and dynamic stability using the Euler-Bernoulli beam theory. The influences of the spinning speed of a twisted Timoshenko beam spinning about its longitudinal axis on the bending frequencies and buckling loads were studied by Liao and Dang [24], Chen and Keer [25] and Chen [26]. The effects of twist angles and rotational speed on the vibrational characteristics were investigated by the finite element method.

The effect of the axial loading on the vibration frequencies and stability of pre-twisted beams has been treated by many scientists and engineers. Based on the Euler-Bernoulli beam theory, the lateral vibration of a spinning twisted beam under a constant axial load was developed by Tekinalp and Ulsoy [19,20] to investigate its influence on the natural frequencies of the drill bit. The bending-bending vibration of a spinning twisted Timoshenko beam subjected to a time-independent axial load was also established by Liao and Dang [24], and Chen and Keer [25] in the spinning twist frame to study the effects of the axial force on the natural frequencies. By directly employing the flexural vibration equations of motion derived by Chen and Keer [25], Chen and Ho [27] used the differential transform method to analyze the natural frequencies and mode shapes of a spinning pre-twisted Timoshenko beam under constant axial forces. Lateral vibrations of an axially loaded non-uniform spinning twisted Timoshenko beam were investigated recently by Ho and Chen [28] based on the differential transform method. The dynamic stability of spinning pre-twisted Euler-Bernoulli beams subjected to time-varying axial loads has been studied by Liao and Huang [29], Tan et. al. [30], and Young and Gau [31] to investigated the effects of pretwist angle, spinning speed and static component of axial load on the instability regions. The static and dynamic stability of a rotating pre-twisted Timoshenko beam subjected to an axial periodic load was studied by Sabuncu and Evran [18] using a finite element model.

The parametric instability of spinning pretwisted beams subjected to time-dependent compressive axial loads has been analyzed by many researchers based on the Euler-Bernoulli beam theory, but the dynamic stability of the spinning twisted Timoshenko beams under time-varying compressive axial forces have not yet been studied. Hence, in the present study, lateral bending vibrations of a spinning pretwisted beam subjected to a harmonically time-dependent compressive axial force have been established by using the Timoshenko beam theory and applying Hamilton's principle. The equations of motion of the twisted beam are derived in the spinning twist coordinate frame. Then the finite element method is utilized to discretize the equations of motion into time-dependent ordinary differential equations with gyroscopic terms. Based on the method by Bolotin [32], a set of second-order ordinary differential equations with periodic coefficients of Mathieu-Hill type is formulated to find the boundaries of instability regions. The effects of pretwist angle, spinning speed, static component of axial force, length-to-width ratio and boundary conditiion on the parametric instability of the spinning pretwisted Timoshenko beam are investigated and discussed. It is believed that the present model is valuable for the parametric studies to understand better the various dynamic aspects of the twisted beam affecting its parametric stability behavior.

2. EQUATIONS OF LATERAL VIBRATION

The vibration equations of motion for a spinning pretwisted Timoshenko beam subjected to the axial pulsating load are derived based on Timoshenko beam theory and applying Hamilton's principle [33]. The derivation of these equations is based on assumptions that the shear center of the beam coincides with its mass center; the plane cross-sections originally normal to the neutral axis remain plane due to shear deflection; the beam rotates at a constant spin speed ω_o about its longitudinal axis; and the twist angle per unit length β_o (ϕ/L, ϕ = total twist angle) of the beam is constant.

To obtain the equations of motion of the rotating pretwisted beam in moving twisted coordinates, three coordinate frames are used as given in Fig. 1. Coordinate system XYZ represents the inertial system; coordinate system xyz rotates with a constant spin speed Ω about the Z-axis in the inertial frame; and the rotating

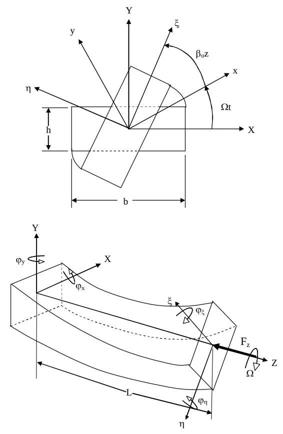

Fig. 1 Beam configuration and coordinate systems.

twist coordinate $\xi\eta z$ is fixed to the rotating beam and moves along the beam pretwist angle such that axes ξ and η are in the principal directions of the beam cross-section.

The transverse vibration equations of motion of the Timoshenko beam in two orthogonal directions are derived by applying Hamilton's principle to the Lagrangian (L) of the beam system, which leads to

$$\delta\int_{t0}^{t1} L dt = \delta\int_{t0}^{t1} (T - V + W) dt = 0 \tag{1}$$

Here T is the total kinetic energy of the beam vibrating in combined bending-bending including the rotating inertia effects; V is the potential energy of the beam due to the shear and bending deformations; W is the work produced by the time-dependent axial force. They are expressed in the inertial coordinate system XYZ as follows. The prime and the dot denote the differentiation with respect to the spatial variable Z and time t, respectively.

$$T = \frac{1}{2}\int_0^L [m(\dot{u}_X^2 + \dot{u}_Y^2)^2 + J_{XX}\dot{\varphi}_X^2 + 2J_{XY}\dot{\varphi}_X\dot{\varphi}_Y + J_{YY}\dot{\varphi}_Y^2] dZ \tag{2}$$

$$V = \frac{1}{2}\int_0^L \{\kappa GA[(u'_X - \varphi_Y)^2 + (u'_Y - \varphi_X)^2] + EI_{XX}(\varphi'_X)^2 + 2EI_{XY}\varphi'_X\varphi'_Y + EI_{YY}(\varphi'_Y)^2\} dZ \tag{3}$$

$$W_1 = -\int_0^L F_Z(t)[(u'_X)^2 + (u'_Y)^2] dZ \tag{4}$$

The relationship between the total transverse displacements, u_X and u_Y, and angles of rotation due to bending, φ_X and φ_Y, in the inertial frame and the associated quantities u , u , φ_ξ and φ_η in the rotating twist coordinate system can be expressed as the following transformation.

$$\begin{Bmatrix} u_X \\ u_Y \\ -\varphi_X \\ \varphi_Y \end{Bmatrix} = \begin{bmatrix} Cos\psi & -Sin\psi & 0 & 0 \\ Sin\psi & Cos\psi & 0 & 0 \\ 0 & 0 & Cos\psi & -Sin\psi \\ 0 & 0 & Sin\psi & Cos\psi \end{bmatrix} \begin{Bmatrix} u_\xi \\ u_\eta \\ -\varphi_\xi \\ \varphi_\eta \end{Bmatrix} \tag{5}$$

where $\psi = \Omega t + \beta_o z$. Similarly, the area moments and product of inertia I_{XX}, I_{YY} and I_{XY} can be expressed in terms of the principal area moments of inertia I_ξ and I_η as follows.

$$\begin{bmatrix} I_{XX} & I_{XY} \\ I_{XY} & I_{YY} \end{bmatrix} = \begin{bmatrix} Cos\psi & -Sin\psi \\ Sin\psi & Cos\psi \end{bmatrix} \begin{bmatrix} I_{\xi} & 0 \\ 0 & I_{\eta} \end{bmatrix} \begin{bmatrix} Cos\psi & -Sin\psi \\ Sin\psi & Cos\psi \end{bmatrix}^T \qquad (6)$$

and by changing the symbol “I” to “J”, Eq. (6) can also be applied to mass moments of inertia. By utilizing the transformations (5)-(6) into Eqs. (1)- (4), the system equations for a spinning pretwisted Timoshenko beam in the rotating twist frame throughout the domain and the associated boundary conditions could be obtained. The governing equations of motion and boundary conditions could be expressed in matrix form as follows.

$$\overline{\mathbf{M}}\ddot{\mathbf{d}} + \overline{\mathbf{C}}\dot{\mathbf{d}} + \overline{\mathbf{K}}_{\omega}\mathbf{d} + \overline{\mathbf{K}}_{1}\mathbf{(t)}\mathbf{d}'' + (\overline{\mathbf{K}}_{2}(t) + \overline{\mathbf{K}}_{3}\mathbf{(t)})\mathbf{d}' + \overline{\mathbf{K}}_{4}\mathbf{(t)}\mathbf{d} = 0 \qquad (7)$$

$$(\overline{\mathbf{K}}_{1}\mathbf{(t)}\mathbf{d}' + \overline{\mathbf{K}}_{2}\mathbf{(t)}\mathbf{d})\delta\mathbf{d}\Big|_{0}^{L} = 0 \qquad (8)$$

Here $\overline{\mathbf{M}}$, $\overline{\mathbf{C}}$ and $\overline{\boldsymbol{K}}_{i}\boldsymbol{(t)}$ are coefficient matrices relating to the system characteristics of the spinning twisted beam structure; $\mathbf{d}$ = [u , u , φ_ξ, φ_η]^T is the displacement matrix of the twisted beam in the rotating twisted frame; u , u , φ_ξ and φ_η are transverse displacements and angles of rotation.

3. DYNAMIC STABILITY ANALYSIS

To investigate the dynamic stability of a spinning twisted Timoshenko beam subjected to an axial pulsating force, a finite element method is utilized to write the governing equations in a discrete form of ordinary differential equations with time as the independent variable. The Mindlin-type linear beam element [34] with eight degrees of freedom is used here. Each node of the beam element has four degrees of freedom, two total transverse displacements and two rotations due to bending. Hence, the displacement function of the beam element can be approximated by

$$\mathbf{d}^{(e)}(z,t) = \mathbf{N}(z)\mathbf{p}^{(e)}(t) \qquad (9)$$

where

$$\boldsymbol{d}^{(e)} = [\,u_{\xi},\ \ u_{\eta},\ \ \varphi_{\xi},\ \ \varphi_{\eta}\,]^T$$

$$\boldsymbol{p}^{(e)} = [\,u_{\xi 1},\ \ u_{\eta 1},\ \ \varphi_{\xi 1},\ \ \varphi_{\eta 1},\ \ u_{\xi 2}\ \ u_{\eta 2},\ \ \varphi_{\xi 2},\ \ \varphi_{\eta 2}\,]^T$$

$$\mathbf{N} = \begin{bmatrix} N_1 & 0 & 0 & 0 & N_2 & 0 & 0 & 0 \\ 0 & N_1 & 0 & 0 & 0 & N_2 & 0 & 0 \\ 0 & 0 & N_1 & 0 & 0 & 0 & N_2 & 0 \\ 0 & 0 & 0 & N_1 & 0 & 0 & 0 & N_2 \end{bmatrix}$$

Here shape functions $N_1 = 1 - z/L_e$ and $N_2 = 1 - z/L_e$, where L_e is the length of the twisted beam element; $u_{\xi 1}$, $u_{\eta 2}$, $\varphi_{\xi 1}$, $\varphi_{\eta 1}$ and $u_{\xi 2}$, $u_{\eta 2}$, $\varphi_{\xi 2}$, $\varphi_{\eta 2}$ are nodal displacements of the twisted beam element at its nodal points.

By using Eq. (9) into the weak form of the equations of motion, (7) and (8), and applying Galerkin‘s criterion, the resulting element equations are obtained as follows

$$\boldsymbol{M}^{(e)}\ddot{\boldsymbol{p}}^{(e)} + \boldsymbol{C}^{(e)}\dot{\boldsymbol{p}}^{(e)} + (\boldsymbol{K}_{\Omega}^{(e)} + \boldsymbol{K}_{B}^{(e)} + \boldsymbol{K}_{F}^{(e)}(t))\boldsymbol{p}^{(e)} = \boldsymbol{0} \qquad (10)$$

Here $M^{(e)}$ and $C^{(e)}$ denote the element inertia and Coriolis gyroscopic matrices; $\boldsymbol{K}_{\Omega}^{(e)} = -\Omega^2\hat{\boldsymbol{M}}^{(e)}$, $\mathbf{K}_{B}^{(e)}$ and $\mathbf{K}_{F}^{(e)} = -F_z\mathbf{(t)}\mathbf{F}^{(e)}$ are the element stiffness matrices due to spinning speed, the bending and shear effects, and axial pulsating force, respectively. The skew-symmetric property of the element Coriolis leads to the typical coupling phenomenon of the behavior of the spinning shaft system in the lateral plane. Special treatment must be taken when deriving the element stiffness of a Timoshenko beam because the numerical integration can cause the shear locking when using elements of C^0-order. The selective reduced integration method proposed by Hughes [35] is used here. The stiffness matrix due to bending effect is computed using the normal quadrature rule, while the stiffness matrix due to shear effect is integrated with one-point quadrature scheme to avoid the shear locking. Directly assembling these element matrices and imposing the essential boundary conditions obtain as the general form of the global system of equations

$$\boldsymbol{M}\ddot{\boldsymbol{p}} + \boldsymbol{C}\dot{\boldsymbol{p}} + (\,\boldsymbol{K}_{\Omega} + \boldsymbol{K}_{B} + \boldsymbol{K}_{F}\,)\boldsymbol{p} = \boldsymbol{0} \qquad (11a)$$

or

$$\boldsymbol{M}\ddot{\boldsymbol{p}} + \boldsymbol{C}\dot{\boldsymbol{p}} + (\,\boldsymbol{K}_{B} + \boldsymbol{K}_{\Omega}\ \text{-}\ F_{\boldsymbol{z}}\boldsymbol{(t)}\boldsymbol{F}\,)\boldsymbol{p} = \boldsymbol{0} \qquad (11b)$$

where the matrices M, C, K_Ω, K_B and K_F are the corresponding global matrices. Eq. (11) is the standard form of a gyroscopic type system of equations of motion.

The periodic compressive axial force is assumed to be of the form

$$F_z = F_o + F_p\cos\omega t = P_{cr}(\alpha + \beta\cos\omega t) \qquad (12)$$

where F_o is the static axial load; F_p and ω are the amplitude of the perturbed axial force and the disturbing frequency; P_{cr} is the critical static buckling load of the non-spinning untwisted beam; α and β are the respective static and dynamic load factors. Introducing Eq. (12) into Eq. (11) yields

$$\boldsymbol{M}\ddot{\boldsymbol{p}} + \boldsymbol{C}\dot{\boldsymbol{p}} + (\,\boldsymbol{K}_{B} + \boldsymbol{K}_{\Omega}\ \text{-}\ \alpha P_{cr}\boldsymbol{F}\ \text{-}\ \beta P_{cr}\cos\omega t\boldsymbol{F}\,)\boldsymbol{p} = \boldsymbol{0} \qquad (13)$$

which represents a second-order ordinary differential equation with periodic coefficients of Mathieu-Hill type. Based on the method presented in Bolotin [32], the boundaries between stable and unstable solutions of Eq. (13) are separated by periodic solutions of periods T and 2T, where T = $2\pi/\omega$. The instability zones bounded by solutions with period 2T are of greater practical importance. As the first-order approximation, the periodic solutions of p with period 2T can be sought in the one-term Fourier series as

$$p = a\sin\frac{\omega t}{2} + b\cos\frac{\omega t}{2} \qquad (14)$$

where a and b are arbitrary time-invariant vectors. By substituting the one-term series solution into equation (13) and separating the sine and cosine parts, a set of homogenous linear algebraic equations in terms of a and b can be obtained in matrix form as follows.

$$\begin{bmatrix} \mathrm{K_B} + \mathrm{K_\Omega} - (\alpha - \frac{1}{2}\beta)P_{cr}\mathrm{F}\ \text{-}\ \frac{\omega^2}{4}\mathrm{M} & -\frac{1}{2}\omega\mathrm{C} \\ \frac{1}{2}\omega\mathrm{C} & \mathrm{K_B} + \mathrm{K_\Omega} - (\alpha + \frac{1}{2}\beta)P_{cr}\mathrm{F}\ \text{-}\ \frac{\omega^2}{4}\mathrm{M} \end{bmatrix} \begin{Bmatrix} \mathrm{a} \\ \mathrm{b} \end{Bmatrix} = 0 \qquad (15)$$

Eq. (15) can be rewritten in the form of quadratic eigenvalue problem (QEP)

$$(\omega^2\mathbf{M_E}+\omega\mathbf{C_E}+\mathbf{K_E})\mathbf{q}=0 \tag{16}$$

with

$$\mathbf{q}=\{\mathbf{a} \quad \mathbf{b}\}^t \tag{17}$$

$$\mathbf{M_E}=\frac{1}{4}\begin{bmatrix}\mathbf{M} & \mathbf{O}\\ \mathbf{O} & \mathbf{M}\end{bmatrix} \tag{18}$$

$$\mathbf{C_E}=\frac{1}{2}\begin{bmatrix}\mathbf{O} & \mathbf{C}\\ \mathbf{-C} & \mathbf{O}\end{bmatrix} \tag{19}$$

$$\boldsymbol{K_E}=\begin{bmatrix}-\boldsymbol{K_B}-\boldsymbol{K}_\Omega+(\alpha-\frac{1}{2}\beta)P_{cr}\boldsymbol{F} & \boldsymbol{O}\\ \boldsymbol{O} & -\boldsymbol{K_B}-\boldsymbol{K}_\Omega+(\alpha+\frac{1}{2}\beta)P_{cr}\boldsymbol{F}\end{bmatrix} \tag{20}$$

In order to determine the eigenvalues of the quadratic eigenvalue problem Eq. (16), a standard linearization approach is used and a 2n x 2n general eigenvalue problem is solved. Since Eq. (16) is of a real gyroscopic system, a Hamiltonian/ skew-Hamiltonian linearization [36] is used to form the 2n x 2n general eigenvalue problem as

$$(\mathbf{A}-\omega\mathbf{B})\mathbf{r}=0 \tag{21}$$

where

$$\mathbf{r}=\{\omega\mathbf{q} \quad \mathbf{q}\}^t \tag{22}$$

$$\mathbf{A}=\begin{bmatrix}\mathbf{O} & -\mathbf{K_E}\\ \mathbf{M_E} & \mathbf{O}\end{bmatrix} \tag{23}$$

$$B=\begin{bmatrix}\mathbf{M_E} & \mathbf{C_E}\\ \mathbf{O} & \mathbf{M_E}\end{bmatrix} \tag{24}$$

The condition for non-trivial solutions of Eq. (21) is

$$\det(\mathbf{A}-\omega\mathbf{B})=0 \tag{25}$$

Here, the eigenvalues of the above equation are just the disturbing frequencies ω for the boundaries of the regimes of instability. In the next, the LAPACK [37] generalized nonsymmetric eigenvalue routines are used to solve the eigenvalue problem.

4. RESULTS AND DISCUSSIONS

The first four parametric instability regions are plotted for an axially loaded pretwisted Timoshenko beam to consider the effects of pretwist angle, spinning speed, static load factor, aspect ratio and boundary conditions. A clamped-free non-spinning untwisted beam with geometric and material properties of L = 0.2 m, b = 0.02 m, R = I_ξ/I_η = 0.4, κ = 5/6, E = 207 Gpa, ν= 0.3, ρ = 7860 kg/m^3 is described as a standard case and the evaluated fundamental natural frequency ω_{cr} and buckling load P_{cr} of this beam is taken as the reference speed and load. The non-dimensional spinning speed $\overline{\Omega}=\Omega/\omega_{cr}$ and boundary frequency ratio $\varpi=\omega/\omega_{cr}$ is used throughout the dynamic instability studies.

The effect of the pretwist angle β_o on the instability regions for a spinning clamped-free pretwisted Timoshenko beam with $\overline{\Omega}=0.4$ and α = 0.5 is shown in Fig. 2. It can be found that the first two instability regions move close to each other as the pretwist angle is increased. The next two instability regions also move close to each other as the pretwist angle is increased up to β_o = 9 cm^{-1}, but they move away from each other when β_o = 13.5 cm^{-1}and they both shift to the left when β_o = 18 cm^{-1}. It can also be observed that only the first instability region become more and more narrow with the increasing pretwist angle, but the other three instability regions vary differently when the pretwist angle is increased.

Fig. 3 shows the effect of the spinning speed $\overline{\Omega}$ on the instability regions for a spinning clamped-free pretwisted Timoshenko beam with β_o = 4.5 cm^{-1} and α = 0.5. When the spinning speed increases, the first two instability regions move away from each other. The next two instability regions also have the same tendency. As can be seen in Fig.3, the first instability region becomes larger and its lower bound is to be shifted to the left and intersects the ordinate axis at a lower dynamic load factor when the spinning speed is increased and near the critical speed. However, the second instability region becomes smaller with the increasing spinning speed. The other instability regions are affected insignificantly by the spinning speed.

The effect of static component of load for α = 0, 0.2, 0.4, 0.6, 0.8 and 1.0 on the instability regions for a spinning clamped-free pretwisted Timoshenko beam with β_o = 4.5 cm^{-1} and $\overline{\Omega}$ = 0.2 is shown in Fig. 4. Due to the increase of static component of the axial load, the instability regions tend to shift to lower frequencies and become wider. It can be seen that the first instability region is significantly affected by the increasing static load factor, but the other instability regions are influence subtly.

Fig. 5 shows the effect of the length-to-width ratio on instability regions for a spinning clamped-free pretwisted Timoshenko beam with β_o = 4.5 cm^{-1}, $\overline{\Omega}$ = 0.4 and α = 0.5. It is observed that the onset of dynamic instability occurs much later with the decreasing of the length-to-width ratio. As the length-to-width ratio decreases, the width of the first and fourth instability region is decreased, but that of the second and third instability region is increased. However, only the first two instability regions are affected significantly by the length-to-width ratio.

Fig. 6 shows the influence of different restrained conditions (clamped- free, clamped- pinned, clamped-clamped) on the instability regions for a spinning pretwisted Timoshenko beam with β_o = 4.5 cm^{-1}, $\overline{\Omega}$ = 0.4 and α = 0.5. As expected, the dynamic instability occurs at a higher disturbance frequency from the free to clamped end. The width of the instability regions are also decreased with the increasing restraint.

5. CONCLUSION

The present Timoshenko beam model can be used to analyze the parametric instability characteristics of a spinning axially-loaded pre-twisted beam structures. The finite element results demonstrate the influence of the twist angle, spinning speed, static load component, length-to-width ratio and restraints on the instability regions. Based on the results discussed earlier, several conclusions can be summarized as follows.

(1) The first instability region becomes smaller, but the next three instability regions change differently as the pretwist angle is increased.
(2) The width of the first instability region becomes wider, but that of the second instability region becomes narrower when the spinning speed increases. The onset of first instability region occurs at a lower disturbance frequency, but that of second instability region occurs at a higher disturbance frequency with the increasing speed.
(3) The instability regions are to be shifted to lower frequencies with wide instability regions when the static load component is increased, which shows a destabilizing effect on the dynamic stability behavior of the twisted beam.
(4) The onset of dynamic instability occurs much later when the length-to-width ratio is decreased. The width of the first instability region is decreased, but that of the second is increased with the decreasing length-to-width ratio.
(5) The instability regions have been influenced due to constraints provided at the ends

6. ACKNOWLEDGEMENT

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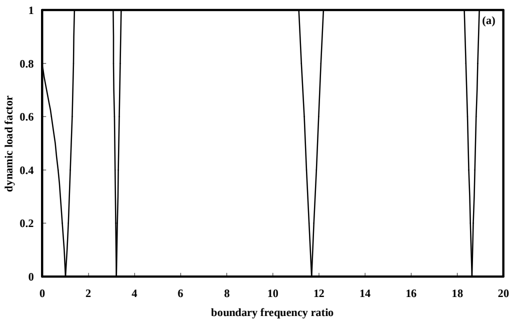


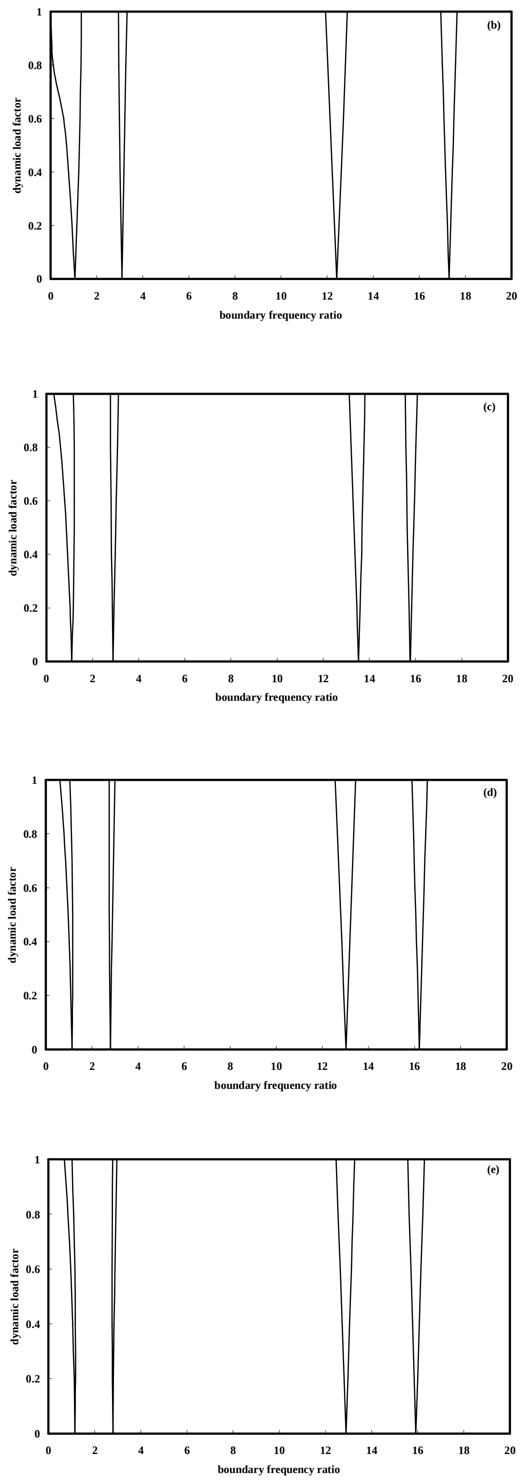
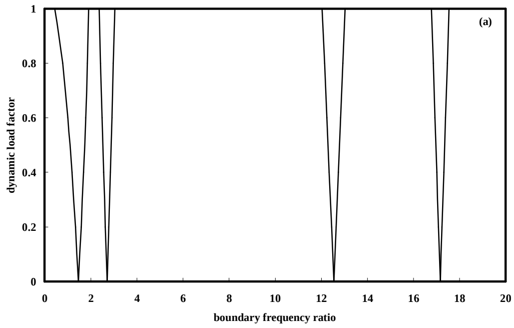
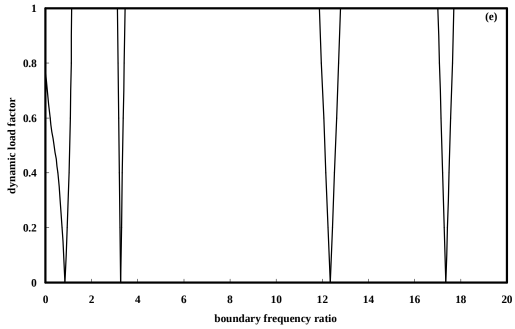
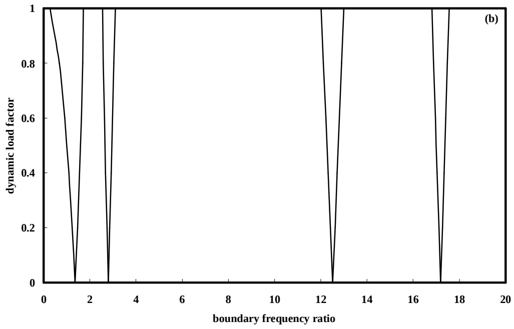
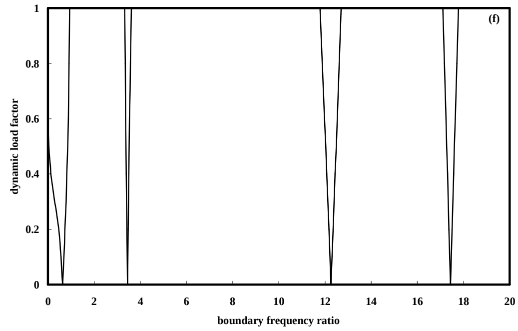


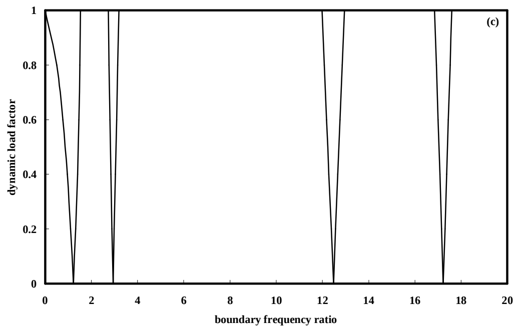
Fig. 2 Effect of pretwist angle β_o on instability regions of a spinning clamped-free pretwisted Timoshenko beam with R = 0.4, $\overline{\Omega} = 0.4$ and α = 0.5; (a) β_o = 0, (b) β_o = 4.5, (c) β_o = 9.0, (d) β_o = 13.5, (e) β_o = 18.











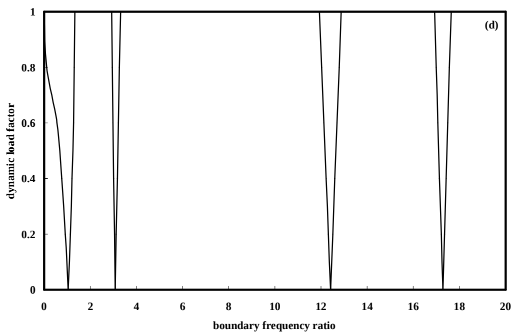
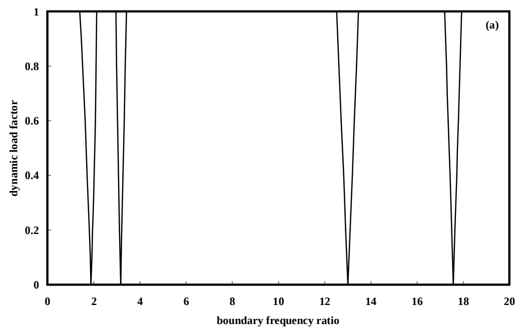
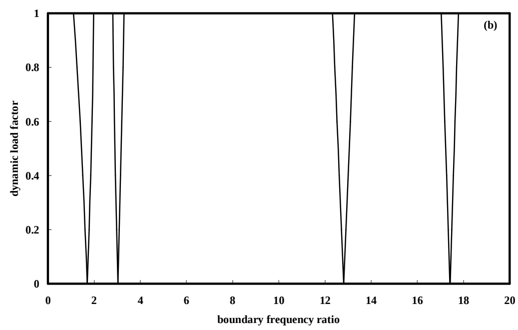


Fig. 3 Effect of spinning speed $\overline{\Omega}$ on instability regions of a spinning clamped-free pretwisted Timoshenko beam with R = 0.4, β_o = 4.5 cm^{-1} and α = 0.5; (a) $\overline{\Omega}$ = 0.1, (b) $\overline{\Omega}$ = 0.2, (c) $\overline{\Omega}$ = 0.3, (d) $\overline{\Omega}$ = 0.4, (e) $\overline{\Omega}$ = 0.5, (f) $\overline{\Omega}$ = 0.6.





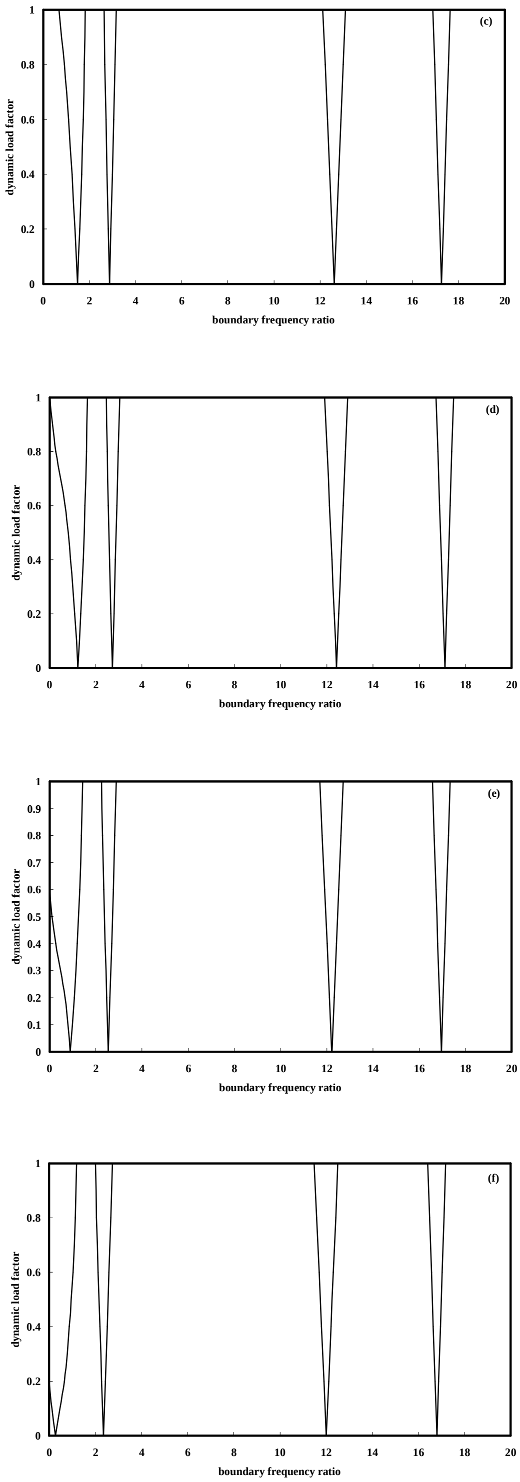


Fig. 4 Effect of static load factor α on instability regions of a spinning clamped-free pretwisted Timoshenko beam with R = 0.4, β_o = 4.5 cm^{-1} and $\overline{\Omega}$ = 0.2; (a) α = 0., (b) α = 0.2, (c) α = 0.4, (d) α = 0.6, (e) α =0.8, (f) α = 1.0.

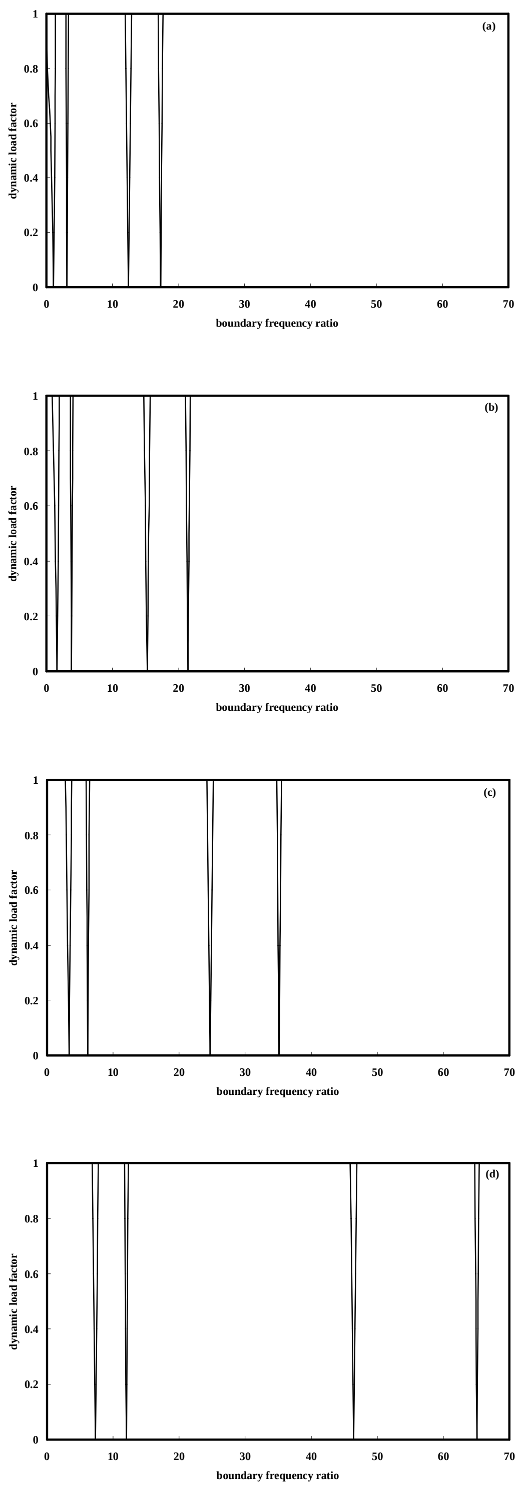


Fig. 5 Effect of length-to-width ratio L/b on instability regions of a spinning clamped-free pretwisted Timoshenko beam with R = 0.4, β_o = 4.5 cm^{-1}, $\overline{\Omega}$ = 0.4 and α = 0.5; (a) L/b = 10, (b) L/b = 9, (c) L/b = 7, (d) L/b = 5.

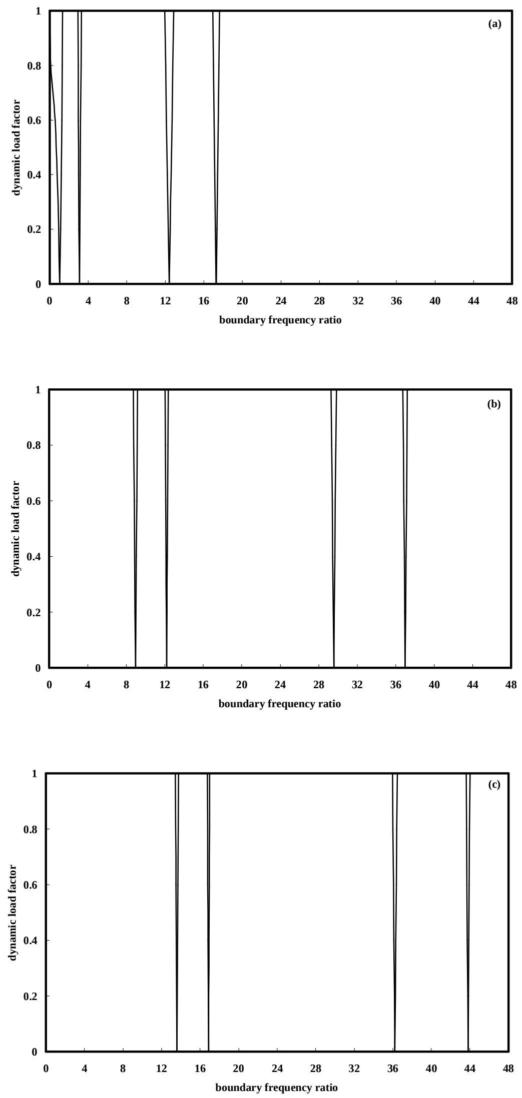


Fig. 6 Effects of restrained conditions on instability regions of a spinning pretwisted Timoshenko beam with R = 0.4, β_o = 4.5 cm^{-1}, $\overline{\Omega}$ = 0.4 and α = 0.5; (a) clamped- free, (b) clamped- pinned, (c) clamped-clamped.

自旋預扭提摩新格樑受周期性變動軸向壓力作用下之參數不穩定分析

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摘要

本計畫使用提摩新格樑理論及漢彌爾敦原理，以建立受到一週期性軸向壓力之自旋預扭樑的撓曲振動方程式。並在轉動扭轉座標系下推導該樑之運動方程式，再使用有限元素法將運動方程式離散成以時間為自變數之陀螺儀型態常微分方程式，以進行動態分析。文中考慮受到週期性變動軸向壓力作用下自旋預扭提摩新格樑的參數不穩定性， Mathieu-Hill 型態之具週期參數的線性二階常微分運動方程式將被形成以探討樑之預扭角、轉速、穩態軸向力、幾何參數比及邊界條件對該系統的不穩定區域的影響。咸信本計畫的分析將有助於更加了解預扭樑的各種參數對其動態穩定行為的影響效應。

關鍵詞：預扭樑、提摩新格樑、陀螺儀、參數不穩定性。