中國文化大學 98 學年度轉學招生考試

系組：機械工程學系三年級　　　日期節次：7 月 29 日第 3 節 13:30-14:50

科目：**工程數學**　　(103-139)

1. Find the general solution of the differential equation $y + xy' = 0$. (10)

2. Given $y'' + 11y' + 24y = 0$; $y_1(x) = e^{-3x}$; $y_2(x) = e^{-8x}$

   (1) Prove that $y_1(x)$ and $y_2(x)$ are solutions of the differential equation. (6)

   (2) Show that $y_1(x)$ and $y_2(x)$ are linearly independent. (5)

   (3) Write the general solution y(x). (4)

3. Use the Laplace transform to solve the following initial value problem. (10)

   $y' + y = e^{-t}$ $y(0) = 0$

4. Determine whether the given vectors $3\vec{i} + 2\vec{j}$ and $\vec{i} - \vec{j}$ are linearly independent or dependent in R^2. (10)

5. Find the general solution of the following nonhomogeneous linear system. (15)

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & -1 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

6. Given a square matrix $A = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$.

   (1) Find its eigenvalues and the associated eigenvectors. (10)
   (2) Show that matrix **A** is nonsingular. (5)
   (3) Find a orthogonal matrix **P** that diagonalizes matrix **A**. (5)

7. Find the streamlines of the vector field $\vec{F}(x,y,z) = 3x^2\vec{i} - y\vec{j} + z^3\vec{k}$. (10)

8. Given a scalar function $\varphi(x,y,z) = xyz$.

   (1) Compute $\nabla\varphi$. (5)
   (2) Compute $curl(\nabla\varphi)$. (5)