中國文化大學 99 學年度轉學招生考試

系組：機械工程學系三年級　　日期節次：7 月 28 日第 3 節 13:30-14:50

科目：**工程數學**　(95-129)

1. Solve the following initial value problem. (15)

$$y' = 2x + 2; \quad y(-1) = 1$$

2. Find the general solution of the following differential equation. (15)

$$y'' - 3y' + 2y = 10\sin(x)$$

3. Find the given Laplace transform and inverse Laplace transform

(1) $L[\cos^2(t)]$ (7)

(2) $L^{-1}\left[\frac{3}{s-7} + \frac{1}{s^2}\right]$ (6)

4. Given two vectors $\vec{F} = -4\vec{i} - 2\vec{j} + 3\vec{k}$ and $\vec{G} = 6\vec{i} - 2\vec{j} - \vec{k}$.

(1) Evaluate $\vec{F} \bullet \vec{G}$. (5)

(2) Evaluate $\vec{F} \times \vec{G}$. (5)

(3) 求由向量 $\vec{F}$ 及 $\vec{G}$ 形成之平行四邊形的面積。 (5)

(4) 證明 $\vec{F}$ 及 $\vec{G}$ 是否為平行，說明理由。 (5)

5. Given a square matrix $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$.

(1) Find its eigenvalues and associated eigenvectors. (10)

(2) Find a matrix **P** that diagonalizes matrix **A**. (3)

(3) $\mathbf{P^{-1}AP}$ = ? (3)

(4) Is matrix **A** orthogonal? Why? (3)

(5) Is matrix **A** symmetric? Why? (3)

(6) Show that matrix **A** is nonsingular. (3)

6. Given a vector field $\vec{F}(x,y,z) = \vec{i} - y^2\vec{j} + z\vec{k}$ and a scalar field $\varphi(x,y,z) = -x^3yz^2$.

(1) Compute $div(curl\vec{F})$. (6)

(2) Compute $div(\nabla\varphi)$. (6)