

1. Find the general solution $y(x)$ of the following differential equation. (20%)

$$(-3x + y + 6)dx + (x + y + 2)dy = 0$$

2. Find the general solution $y(x)$ of the following differential equation. (20%)

$$\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 3y = \sin 2x$$

3. Find the general solution $y(x)$ of the following differential equation. (20%)

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = x$$

4. Solve the following initial value problem by Laplace transform only. (20%)

$$\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 4y = 0, y(0) = 2.1, y'(0) = 3.9$$

5. Solve the following partial differential equation where L is constant. (20%)

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$\text{Boundary conditions: } u(0, t) = 0, u(L, t) = 0$$

$$\text{Initial conditions: } u(x, 0) = \begin{cases} 1, & 0 < x < L/2 \\ 0, & L/2 < x < L \end{cases}$$