

中國文化大學 101 學年度暑假轉學招生考試

系組：應用數學系三年級 日期節次：7 月 24 日第 2 節 10:50-12:10

科目：高等微積分

U-3-1

1. (15%) Prove that $\sum_{n=1}^{\infty} \sin\left(\frac{x}{n^3}\right)$ converges uniformly on any bounded interval on \mathbb{R} .

2. (20%) Suppose that $\lim_{n \rightarrow \infty} x_n = 2$. Prove that

$$\lim_{n \rightarrow \infty} \frac{3(x_n)^2 + 2}{7x_n} = 1 \text{ by Definition.}$$

3. (15%) Suppose that $0 < a < b$ and $n \in \mathbb{N}$ is odd

$$\text{If } f \text{ is continuous on } [a, b] \text{ and } \int_a^b f(x)x^n dx = 0,$$

prove that $f(x) = 0$ for at least one $x \in [a, b]$.

4. (15%) Let $\sum_{k=1}^n k \cdot a_k = \frac{n+1}{n+2}$ for $\forall n \in \mathbb{N}$. Find $\sum_{n=1}^{\infty} a_n$.

5. (15%) Prove that $f(x) = x^3$ is not uniformly continuous in $(0, \infty)$.

6. (20%) Prove that the following limit exists and evaluates it.

$$\lim_{n \rightarrow \infty} \int_1^3 \frac{nx^{67} + 5}{x^3 + nx^{66}} dx.$$