

0-3-1

一、單選題，每題 3 分，共 30 分。(請將答案依照題號寫在答案紙上，不需寫出計算過程。)

1. () Us the fact that $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ to find the value of $\lim_{x \rightarrow 0} \frac{2x}{\tan 7x}$.
- (A) ∞ (B) 0 (C) 1 (D) $\frac{2}{7}$ (E) $\frac{7}{2}$.
2. () $\int_{-\infty}^{\infty} x e^{-x^2} dx =$ (A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{-1}{2}$ (E) None of these.
3. () The slope of the normal (法線斜率) to the curve $x^3 + y^3 - 6xy = 1$ at the point $P(0, 1)$ is
- (A) 1 (B) -1 (C) 2 (D) $\frac{-1}{2}$ (E) None of these.
4. () The graph of the polar equation $r^2 = 4r \cos \theta$ is a
- (A) cardioids(心臟線) (B) line(直線) (C) spiral(螺線) (D) roses(玫瑰線) (E) circle(圓).
5. () The volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$, $0 \leq x \leq 4$ and the x -axis is (A) $\frac{16\pi}{3}$ (B) 2π (C) 8π (D) $\frac{\pi}{4}$ (E) None of these.
6. () If f is continuous on $[0, 1]$, then $\int_0^1 f(x) dx =$
- (A) $\int_0^1 f(1-x) dx$ (B) $\int_0^1 f(1+x) dx$ (C) $\int_0^1 f(x-1) dx$ (D) $\int_1^0 f(x-1) dx$ (E) None of these.
7. () Which of the following series converges?
- (A) $\sum_{n=1}^{\infty} \frac{1}{n^2 + n + 1}$ (B) $\sum_{n=1}^{\infty} 2^n$ (C) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ (D) $\sum_{n=2}^{\infty} \frac{1}{\ln n}$ (E) None of the above.
8. () $\int_0^2 \sqrt{4-x^2} dx = ?$ (A) $\frac{\pi}{2}$ (B) π (C) 2π (D) $\frac{2\pi}{3}$ (E) 4.
9. () If $f(x) = x^5 + 2x + 1$, then $(f^{-1})'(4) =$ (A) $\frac{1}{5}$ (B) $\frac{1}{6}$ (C) $\frac{1}{7}$ (D) $\frac{1}{8}$ (E) $\frac{1}{9}$.
10. () The average value of $f(x) = 1 + x^2$ on the interval $[-1, 2]$ is
- (A) 2 (B) 3 (C) 4 (D) 6 (E) None of these

中國文化大學 100 學年度轉學招生考試

系組：應用數學系二年級

日期節次：7 月 26 日第 1 節 09:00-10:20

科目：微積分 (122-22)

二、填充題，每格 5 分，共 40 分。(請將答案依照題號寫在答案紙上，不需寫出計算過程。)

1. $\lim_{x \rightarrow 0} \frac{|x|+3x}{2x} = \underline{\hspace{2cm}}$.

2. $\frac{d}{dx} \int_{\sqrt{x}}^1 \sin(t^2) dt = \underline{\hspace{2cm}}$.

3. $\int_0^3 x\sqrt{1+x} dx = \underline{\hspace{2cm}}$.

4. $\int_0^1 \int_x^1 \sin(y^2) dy dx = \underline{\hspace{2cm}}$.

5. The area of the region enclosed by $x = y^2$ and $y = x - 2$ is $\underline{\hspace{2cm}}$.6. The slope of the cycloid $x = 2(t - \sin t)$, $y = 2(1 - \cos t)$ at the point where $t = \frac{\pi}{3}$ is $\underline{\hspace{2cm}}$.7. The interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{4^n x^{2n}}{n}$ is $\underline{\hspace{2cm}}$.8. The directional derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P_0(1, 1, 0)$ in the direction $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$ is $\underline{\hspace{2cm}}$.

三、計算證明題，每題 10 分，共 30 分。(請寫下計算證明過程，否則不予計分。)

1. Use $\varepsilon - \delta$ method to prove $\lim_{x \rightarrow 3} x^2 = 9$.2. Find all relative extrema and saddle points of $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$.3. Evaluate the integral $\int_0^1 \int_0^{1-x} \sqrt{x+y}(y-2x)^2 dy dx$. (Hint: Let $u = x + y$, $v = y - 2x$)

第 2 頁共 2 頁

本 試 題 採
雙 面 印 刷