

中國文化大學八十四學年度研究所碩士班入學考試

所(組)別： 國際企業管理研究所

考試科目： 微 積 分

1. (10%) Determine a polynomial function F satisfying

$$F'(x) - F(x) = x^3.$$

2. (10%) Prove that

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n^2}\right)^n = 1.$$

3. (10%) If f is a continuous function in a closed interval $[a, b]$ and if f' contains the open interval (a, b) in its domain, please prove that there exists a number c in (a, b) such that $f(b) - f(a) = (b - a)f'(c)$.

4. (10%) Evaluate

$$\int (x^2 - 2) \exp(x^3 - 6x) dx.$$

5. (10%) Verify that

$$\int e^{x^3} dx = \frac{e^{x^3}}{3x^2} + \frac{2}{3} \int \frac{e^{x^3}}{x^3} dx.$$

6. (10%) A company's profit (in dollars) from the sale of x VCRs is approximately

$$P(x) = \frac{10x^2 - 50x}{x + 1}$$

- (a) Determine the marginal profit function.
 (b) What is the limit of the profit as the quantity sold increases without bound?

中國文化大學八十四學年度研究所碩士班入學考試

所(組)別： 國際企業管理研究所

考試科目： 微 積 分

7. (10%) A company determines that the value of an investment is V , in millions of dollars, after time t , in years, where V is given by

$$V(t) = 5t^3 - 30t^2 + 45t + 5\sqrt{t}$$

- (a) Find the average rate of change of the investment between year 1 and year 5.
(b) Find the equation of the tangent line to the graph at the point $(1, f(1))$.
8. (10%) The total sales S of a one-product firm are given by $S(L, M) = ML - L^2$, where M = the cost of materials and L = the cost of labor. Find the maximum value of this function subject to the budget constraint $M + L = 80$.
9. (10%) A firm produces two kinds of radio, one that sells for \$17 each and the other for \$21 each. The total revenue from the sale of x thousand radios at \$17 each and y thousand at \$21 each is given by $R(x, y) = 17x + 21y$. The company determines that the total cost, in thousands of dollars, of producing x thousand of the \$17 radio and y thousand of the \$21 radio is given by $c(x, y) = 4x^2 - 4xy + 2y^2 - 11x + 25y - 3$. Find the amount of each type of radio that must be produced and sold in order to maximize profit.
10. (10%) A television company determines that the marginal cost of producing the x th TV is given by $C'(x) = 100 - 0.2x$, $C(0) = 0$. It also determines that its marginal revenue from the sale of the x th TV is given by $R'(x) = 100 + 0.2x$, $R(0) = 0$. Find the total profit from the production and sale of 1000 TVs.