

系所(組)別：數位機電科技研究所

考試科目：工程數學

M-7-2

- (1) A, B are two matrices, indicate true or false for each of the following statements (no proof is needed) (10%)
- 1.)  $AI=IA$
  - 2.)  $AB=BA$
  - 3.)  $A+B=B+A$
  - 4.)  $A+(B+C)=(A+B)+C$
  - 5.)  $A^{-1}A=AA^{-1}=I$
- (2)  $AX=b$  is a matrix equation, under what condition --the equation is a homogeneous equation? (5%)
- (3) A is a  $n \times n$  complex matrix, under what condition—A will be a Hermitian matrix? (10%)
- (4)  $A = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{bmatrix}$ , find L, U matrix, satisfy  $A=LU$  (10%)
- (5) Prove  $(AB)^{-1} = B^{-1}A^{-1}$  (10%)
- (6) Find inverse of the matrix for  $A = \begin{bmatrix} 1 & 4 & 5 \\ 4 & 2 & 5 \\ -3 & 9 & 8 \end{bmatrix}$  (10%)
- (7) Describe geometrically the subspace of  $R^3$  spanned by (10%)
- (a)  $(0,0,1), (0,1,1), (0,2,1)$
  - (b)  $(0,0,0), (0,2,0), (0,5,0)$
- (8) Find two different bases for the subspace of all vectors in  $R^3$ , where first two components are equal. (10%)
- (9) Given the matrix  $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 6 & 8 \\ 3 & 6 & 7 & 12 \\ 4 & 8 & 12 & 17 \end{bmatrix}$
- (a) Compute the determinant of A
  - (b) Does A have any complex eigenvalues? Explain without computation (10%)
- (10) Find the  $PA=LDU$  factorizations of  $A = \begin{bmatrix} 0 & 1 & 2 & 3 \end{bmatrix}$ , and  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix}$  (P is a permutation matrix) (15%)