中國文化大學 99 學年度碩士班考試入學招生考試

系所組：機械工程學系數位機電碩士班乙組、丙組

日期節次：99 年 3 月 13 日第 1 節 9:00 -10:30

科目：工程數學

1. Verify that the function, $y = e^{-x^2}$, is a solution of the equation, $\frac{dy}{dx} + 2xy = 0$. (10%)

2. Find a general solution of the linear system
   $x' = y$
   $y' = x$ (15%)

3. Explain what the Dirac delta function, $\delta(t-a)$, is. (5%) Find a solution of the mathematical model: $y'' = \delta(t-1)$, $y(0) = 0$, $y'(0) = 0$. (15%)

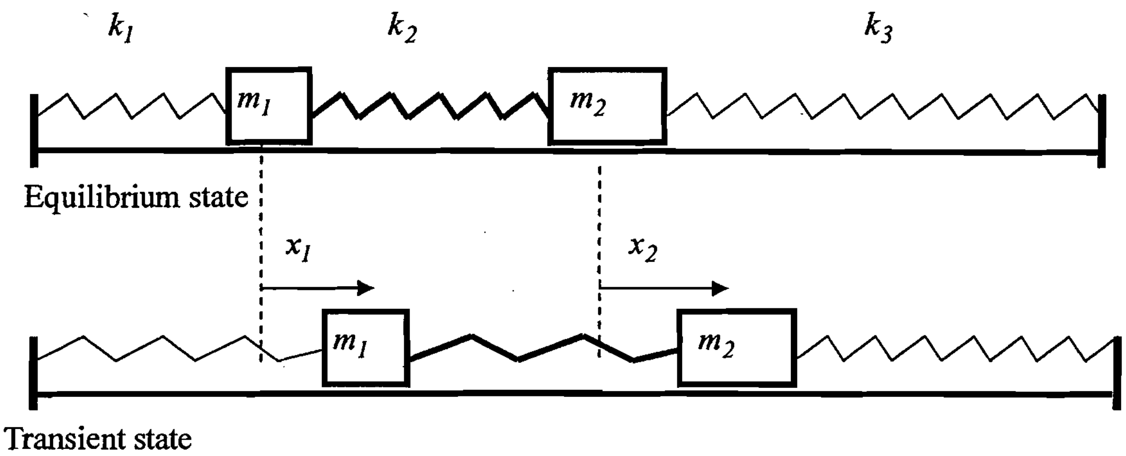
   Some Laplace transforms and general formulas are listed for your reference.

   $L(f') = sL(f) - f(0)$, $L[\delta(t-a)] = e^{-as}$, $L[f(t-a)u(t-a)] = e^{-as}F(s)$, $L(t) = \frac{1}{s^2}$

4. If the inverse of a matrix $\begin{bmatrix} -1 & 1 & a \\ 3 & -1 & 1 \\ b & 3 & 4 \end{bmatrix}$ is $\begin{bmatrix} -0.7 & 0.2 & z \\ x & -0.2 & 0.7 \\ 0.8 & y & -0.2 \end{bmatrix}$, find the values for the elements a and b. (10%)

5. Find the directional derivative of f at P in the direction of $\vec{a}$.
   $f = x^2 + y^2 - z$, $P: (1,1,-2)$, $\vec{a} = [1,1,2]$. (15%)

6. Derive the equations of motion for the two blocks shown in the following figure. $k_i, i = 1,2,3$ are spring constants. $m_i, i = 1,2$ represent the masses of the two blocks. x_i, i=1,2 indicate the displacements of the two blocks from equilibrium. (20%)



6. Derive the mathematical model for the current $i(t)$ in the LC-circuit, assuming zero initial current and charge on the capacitor. (10%)