系所組：化學工程與材料工程學系奈米材料碩士班甲組

日期節次：100年 3 月 19 日第 2 節 11：00-12：30

科目：工程數學

1. (40%) Solve $y(x)$ for the following ordinary differential equation:

   (a) $y'''-y''+100y'-100y=0, y(0)=4, y(0)'=11, y''(0)=-299$ (10%)

   (b) $y'''+3y''+3y'+y=30e^{-x}, y(0)=3, y'(0)=-3, y''(0)=-47$ (10%)

   (c) $x^3y'''-3x^2y''+6xy'-6y=x^4\ln x$, *with* $x>0$ (10%)

   (d) $2xyy'-y^2=-x^2$ (10%) *Hint: use u=y/x*

2. (10%) Derive the Laplace transform for the function {cosh kt} is

   $$\mathcal{L}\{\cosh kt\}=\frac{s}{s^2-k^2}$$

3. (25%) The eigenvalues and normalized eigenvectors of the matrix

   $$\mathrm{M}=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}.$$

4. (25%) Using the Fourier series to expand the following function:

   (a) $f(x)=\cos(ax)$, *with* $-\pi<x\le\pi$ *and* $a\neq$ integer (15%)

   (b) if x=π , please show that : $\cot(x)=\sum_{n=-\infty}^{\infty}\frac{1}{x+n\pi}$, $n=$ integer (10%)

There are some useful formulae.

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}, with\ -L<x\le L$$

$$a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi x}{L}dx,\quad n=0,1,2,3\cdots\qquad b_n=\frac{1}{L}\int_{-L}^{L}f(x)\sin\frac{n\pi x}{L}dx,\quad n=1,2,3\cdots$$

$$2\cos\alpha\cos\beta=\cos(\alpha-\beta)+\cos(\alpha+\beta)$$

$$\sin(a\pm n)\pi=(-1)^n\sin a\pi, if\ n=\text{integer}$$