1(a) 何謂函數正交？(10%)

(b) Legenre Polynomials, $P_2(x) = (3x^2 - 1)/2$, $P_3(x) = (5x^3 - 3x)/2$

證明 $P_2(x)$ 與 $P_3(x)$ 為正交 (10%)

2(a) 函數 $f(x) = \begin{cases} 1, if |x| < 1 \\ 0, if |x| > 1 \end{cases}$，求此函數之 Fourier Integral (10%)

(b)利用 2(a)之結果求 $\int_0^\infty \dfrac{\sin w}{w} dw$ 之積分值 (10%)

3. Reduce to Bessel's equation and find the general solution

(a) $x^2 y'' + xy' + (\lambda^2 x^2 - \nu^2) y = 0$, hint $(\lambda x = z)$ (10%)

(b) $x^2 y'' + xy' + (x^2 - \dfrac{1}{16}) y = 0$ (10%)

4. 求微分方程式之通解

(a) $y'' + y' + y = \sin(x)$ (10%)

(b) $y' = 2\sec(2y)$ (10%)

5. Green's theorem in the plane $\iint\limits_R \nabla \bullet \vec{F} dxdy = \oint \vec{F} \bullet \hat{n} ds$

證明 $\iint\limits_R \nabla^2 w dxdy = \oint \dfrac{\partial w}{\partial n} ds$ (20%)