行政院國家科學委員會專題研究計畫　成果報告

利用統計資料探討在模糊需求和模糊產量下的模糊生產庫存

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行政院國家科學委員會補助專題研究計畫 ■ 成 果 報 告
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執行單位：中國文化大學應用數學系

中　華　民　國　95　年　06　月　27　日

Fuzzy Product Inventory for Fuzzy Demand Quantity and Fuzzy Production Quantity Based on Statistical Data

Abstract
In production inventory, when demand quantity d and production quantity r per day are unknown, how to consider production inventory is a problem. Let demand quantity and production quantity per day of population be D^* and R^* respectively . From the statistical data in the past, we can find the point estimate of D^* and R^*. But we cannot find the probability of the error of the point estimate. We use the $100(1-\alpha)\%$ confidence intervals of D^* and R^* to decive level $(1-\alpha)$ fuzzy numbers $\tilde{d}$ and $\tilde{r}$. Then we get fuzzy total cost and its centroid to represent the estimate of total cost in the fuzzy sense.

Keywords: Fuzzy Set, Fuzzy numbers, Interval-valued fuzzy numbers, Statistical Confidence intervals.

1 Introduction

Yao et al. [1~8]considered fuzzy inventory with or without backorder and fuzzy production inventory problems. In [3,6], they fuzzify order quantity q to triangular fuzzy number and trapezoidal fuzzy number in crisp total cost and obtain fuzzy total cost. Then they use extension principle to find its membership function. Through defuzzification by centroid, they got total cost in the fuzzy sense. In [1], they discussed fuzzy inventory with backorder. They fuzzified backorder quantity s to triangular fuzzy number and kept order quantity q as crisp variable in crisp total cost. Then they used extension principle and centroid to find the total cost in the fuzzy sense. In [5,6], fuzzy inventory with backorder were discussed. The order quantity q was fuzzified to triangular fuzzy numbers and trapezoidal fuzzy number with backorder quantity being kept as crisp variable in crisp total cost. In [2], they discussed fuzzy production inventory. They fuzzified both demand quantity r and production quantity d to triangular fuzzy numbers in crisp total cost . [4] was about fuzzy production inventory . They fuzzified production quantity q per cycle to triangular fuzzy number in crisp total cost. In [7], they discussed fuzzy inventory without backorder and fuzzified both order quantity q and total demand r to triangular fuzzy number in crisp total cost. [8] was about inventory with backorder. They fuzzified total demand r to interval-valued fuzzy set in crisp total cost. The statistical data were not used in the fuzzy inventories stated above. In practical situations, it is more appropriate to use statistical data in the past to solve the problem. In this article, we use statistical data to consider fuzzy production inventory. If we have n groups of data (d_j, r_j), $j = 1,2,...,n$, where d_j and r_j are production quantity and demand per day, then we have confidence intervals $100(1-\alpha)\%$ for production quantity and demand. Corresponding to these intervals, we set level $1-\alpha$ fuzzy numbers $\tilde{d}$ and $\tilde{r}$. Using $\tilde{d}$ and $\tilde{r}$, we can fuzzify crisp total cost to fuzzy total cost. Through extension principle and centroid, we get total cost in the fuzzy sense. We give an example in section 3 and section 4 is the discussion.

2 Membership function of the fuzzy total cost based on statistical data

The inventory control and the production process model is shown in Figure 1. We introduce the following variables.

T : the whole period for the plan (days)

q : quantity produced per cycle

a : holding cost per unit per day

b : production cost per cycle

d : production quantity per day

R : the total demand quantity of whole plan period

r : demand quantity per day

s : the maximal stock quantity

t_s : time for production in each cycle

t_q : time for each cycle

〈Insert Figure 1 here〉

OC and CC' are cycles. O and C are starting points for each cycle. The production begins from O and C with production rate of d units per day, it ends in B and B', and demand rate of r units per day. At B and B', the stock quantity is s (maximal stock quantity). In period $BC(=t_q-t_s)$ and $B'C'$, we only sell without production and demand rate is also r units per day.

Obviously, $t_s=\frac{q}{d}$. Hence $s=q-rt_s=q(1-\frac{r}{d})$ and $\frac{q}{t_q}=\frac{R}{T}$. The stock quantity per cycle is $\frac{s}{2}$.

The times of production during the plan for the whole period T is approximately $\frac{R}{q}$. Then we have total cost $F(q)$ for the whole period T, where

$$F(q)=(\frac{1}{2}at_q s+b)\frac{R}{q}$$
$$=\frac{1}{2}a(1-\frac{r}{d})Tq+\frac{bR}{q} \quad , \quad 0<q \quad (1)$$
$$0<r<d$$

The optimal solution will be attained at $q_*=\sqrt{\frac{2bR}{cT}}$ and the minimal total cost is

$F(q_*)=\sqrt{2bcRT}$, where $c=a(1-\frac{r}{d})$.

In practical situations, r and d are estimates. We don't know the exact value of them. Suppose the production inventory has run through n times in the past. Let the production quantity per day be $d_1,d_2,...,d_n$ and demand quantity per day be $r_1,r_2,...,r_n$, where $0<r_j<d_j$, $j=1,2,...,n$. If

the populations of the production quantity D^* and demand quantity R^* per day are unknown, we can get the point estimate $\bar{d}$ of D^* and the point estimate $\bar{r}$ of R^*, where $\bar{d} = \frac{1}{n}\sum_{j=1}^{n} d_j$ and $\bar{r} = \frac{1}{n}\sum_{j=1}^{n} r_j$. After substituting into (1), we have the following result.

Property 1. Using statistical data d_j and r_j, $j = 1,2,...,n$, we get total cost

$F_0(q) = \frac{1}{2}a(1-\frac{\bar{r}}{\bar{d}})Tq + \frac{bR}{q}, 0 < q$. The optimal solution occurs when the production quantity per

cycle is $q_0 = \sqrt{\frac{2bR}{c_0 T}}$ and the minimal total cost is

$$F_0(q_0) = \sqrt{2bc_0 RT}, \text{ where } c_0 = a(1-\frac{\bar{r}}{\bar{d}}) \tag{2}$$

Since we don't know the probability of the error of the point estimate, we consider the following confidence interval. The $100(1-\alpha)\%$ confidence intervals of D^* and R^* are

$$[\bar{d} - z(\alpha), \bar{d} + z(\alpha)] \tag{3}$$

and

$$[\bar{r} - w(\alpha), \bar{r} + w(\alpha)] \tag{4}$$

respectively, where $z(\alpha) = t_{n-1}(\alpha)\frac{u_1}{\sqrt{n}}$, $w(\alpha) = t_{n-1}(\alpha)\frac{u_2}{\sqrt{n}}$,

$u_1^2 = \frac{1}{n-1}\sum_{j=1}^{n}(d_j - \bar{d})^2$ and $u_2^2 = \frac{1}{n-1}\sum_{j=1}^{n}(r_j - \bar{r})^2$.

If T^* is the *p.d.f.* of *t* distribution with degree *n*-1, then $t_{n-1}(d)$ satisfies $P(|T^*| > t_{n-1}(\alpha)) = \alpha$.

Then

$$P(\bar{d} - z(\alpha) \le D^* \le \bar{d} + z(\alpha)) = 1-\alpha \tag{5}$$

and

$$P(\bar{r} - w(\alpha) \le R^* \le \bar{r} + w(\alpha)) = 1-\alpha \tag{6}$$

Remark 1. In (3), $\bar{d} = \frac{1}{n}\sum_{j=1}^{n} d_j$, d_j, $j = 1,2,...,n$ are considered to be sample values. In (5), d_j, $j = 1,2,...,n$ are considered as random samples (*i.e.* random variables). (4) and (6) have the same arguments.

Let $0 < \alpha_k < 1$, $0 < \beta_k < 1$, $k = 1,2$, $0 < \alpha < 1, 0 < \beta < 1$ and $\alpha_1 + \alpha_2 = 2\alpha$, $\beta_1 + \beta_2 = 2\alpha$

From (5),(6) and the fact that the graph of T^* distribution is symmetric with respect to the *y* axis, we have

$$1-\alpha_k = P(\bar{d} - Z(\alpha_k) \le D^* \le \bar{d} + Z(d_k))$$

$$= 2P(\bar{d} - Z(\alpha_k) \le D^* \le 0)$$

$$= 2P(0 \le D^* \le \bar{d} + Z(\alpha_k)) \quad , \quad k = 1,2$$

$$1-\beta_k = P(\bar{r} - \omega(\beta_k) \le R^* \le \bar{r} + \omega(\beta_k))$$

$$= 2P(\bar{r} - \omega(\beta_k) \le R^* \le 0)$$

$$= 2P(0 \le R^* \le \bar{r} + \omega(\beta_k)) \quad , \quad k = 1,2$$

Then $P(\bar{d} - Z(\alpha_1) \le D^* \le \bar{d} + Z(\alpha_2))$

$$= P(\bar{d} - Z(\alpha_1) \le D^* \le 0) + P(0 \le D^* \le \bar{d} + Z(\alpha_2))$$

$$= \frac{1}{2}(1-\alpha_1) + \frac{1}{2}(1-\alpha_2)$$

$$= 1-\alpha$$

It follows that $[\bar{d} - Z(\alpha_1), \bar{d} + Z(\alpha_2)]$ is the $100(1-\alpha)\%$ confidence interval. Corresponding to the confidence interval, we set level $(1-\alpha)$ fuzzy number

$$\tilde{d} = (\bar{d} - Z(\alpha_1), \bar{d}, \bar{d} + Z(\alpha_2); 1-\alpha) \qquad (7)$$

Its membership function is

$$u_{\tilde{d}}(d) = \begin{cases} \dfrac{(1-\alpha)[d - \bar{d} + Z(\alpha_1)]}{Z(\alpha_1)} & \bar{d} - Z(\alpha_1) \le d \le \bar{d} \\ \dfrac{(1-\alpha)[\bar{d} + Z(\alpha_2) - d]}{Z(\alpha_2)} & \bar{d} \le d \le \bar{d} + Z(\alpha_2) \\ 0 & \text{otherwise} \end{cases} \qquad (8)$$

Remark 2 The grade of membership of $\tilde{d}$ at the point estimate $\bar{d}$ is $1-\alpha$ (maximum). The grades of membership at the two endpoints of the confidence interval are 0. The farther the point from $\bar{d}$ in both sides, the less the grades of its membership. In this $100(1-\alpha)\%$ confidence interval $[\bar{d} - Z(\alpha_1), \bar{d} + Z(\alpha_2)]$, the error between any point in this interval and the point estimate $\bar{d}$ is that it is zero at $\bar{d}$ and it will be maximum at two endpoints. From the pint of view of confidence level, the confidence level at $\bar{d}$ is 1 and the confidence level at the two end points is zero. The farther the point from $\bar{d}$ in both sides, the less of its confidence level. This fact shares the same property as that of grade of membership of $\tilde{d}$. Therefore, the correspondence between $100(1-\alpha)\%$ confidence level $[\bar{d} - Z(\alpha_1), \bar{d} + Z(\alpha_2)]$ and fuzzy number $\tilde{d}$ is reasonable.

Similarly, we consider the $100(1-\alpha)\%$ confidence interval $[\bar{r}-\omega(\beta_1), \bar{r}+\omega(\beta_2)]$ of R^*.
We set level $(1-\alpha)$ fuzzy number

$$\tilde{r} = (\bar{r}-\omega(\beta_1), \bar{r}, \bar{r}+\omega(\beta_2); 1-\alpha) \quad (9)$$

It's membership function is

$$\mu_{\tilde{r}}(r) = \begin{cases} \dfrac{(1-\alpha)(r-\bar{r}+\omega(\beta_1))}{\omega(\beta_1)}, & \bar{r}-\omega(\beta_1) \le r \le \bar{r} \\ \dfrac{(1-\alpha)(\bar{r}+\omega(\beta_2)-r)}{\omega(\beta_2)}, & \bar{r} \le r \le \bar{r}+\omega(\beta_2) \\ 0, & otherwise \end{cases} \quad (10)$$

For any $0<q$, let

$$G_q(d,r) = F(q)$$

$$= \frac{1}{2}a(1-\frac{r}{d})Tq + \frac{bR}{q} \quad (11)$$

We fuzzify *d* in (7) to a level $(1-\alpha)$ fuzzy number $\tilde{d}$ and fuzzify *r* in (9) to a level $(1-\alpha)$ fuzzy number $\tilde{r}$. Then we get fuzzy total cost $G_q(\tilde{d},\tilde{r})$. Through extension principle, we can find its membership function.

Let $G_q(d,r) = z(>0)$, *q*>0.

We have $r = (\dfrac{aTq^2 - 2zq + 2bR}{aTq^2})d$, $0<r<d$. (12)

For any *q*>0, since $0<r<d$, *z* should satisfy the following formula

$$0 < \frac{aTq^2 - 2zq + 2bR}{aTq^2} < 1 \text{ and } z>0$$

Hence $0 < aTq^2 - 2zq + 2bR, -2zq + 2bR < 0$ and $z>0$ (13)

Equivalently, for any *q*>0, we have

$$\frac{bR}{q} < z < \frac{aTq^2 + 2bR}{2q} (= \frac{1}{2}aTq + \frac{bR}{q}) \quad (14)$$

In the following discussion, *z* must satisfy (14)

For any *q*>0, by extension principle,

$$\mu_{G_q(\tilde{d},\tilde{r})}(z) = \sup_{\substack{(d,r)\in G_q^{-1}(Z)\\ 0<r<d}} [\mu_{\tilde{d}}(d) \wedge \mu_{\tilde{r}}(r)]$$

$$= \sup_{0<d} [\mu_{\tilde{d}}(d) \wedge \mu_{\tilde{r}}(\frac{(aTq^2 - 2zq + 2bR)d}{aTq^2})] \quad (15)$$

From (10),

$$\mu_{\tilde{r}}(\frac{(aTq^2 - 2zq + 2bR)d}{aTq^2})$$

$$
= \begin{cases} \dfrac{(1-\alpha)[(aTq^2 - 2zq + 2bR)d - aTq^2(\bar{r} - \omega(\beta_1))]}{aTq^2\omega(\beta_1)} , & P_1 \le d \le P_2 \\ \dfrac{(1-\alpha)[(aTq^2(\bar{r} + \omega(\beta_2)) - (aTq^2 - 2zq + 2bR)d)]}{aTq^2\omega(\beta_2)} , & P_2 \le d \le P_3 \\ 0 , & otherwise \end{cases} \quad (16)
$$

Where $P_1 = \dfrac{aTq^2(\bar{r} - \omega(\beta_1))}{aTq^2 - 2zq + 2bR}$

$$P_2 = \frac{aTq^2\bar{r}}{aTq^2 - 2zq + 2bR}$$

$$P_3 = \frac{aTq^2(\bar{r} + \omega(\beta_2))}{aTq^2 - 2zq + 2bR}$$

Remark 2 z must satisfy (14)

From (8), (15) and (16) we can find $\mu_{G_q(\tilde{d},\tilde{r})}(z)$.

〈Insert Figure 2 here〉

When $P_2 \le \bar{d}$ and $\bar{d} - Z(\alpha_1) \le P_3$, from figure 2, the first formula of (8) and the second formula of (16), we get

$$\frac{(1-\alpha)[d - \bar{d} + Z(\alpha_1)]}{Z(\alpha_1)} = \frac{(1-\alpha)[aTq^2(\bar{r} + \omega(\beta_2)) - (aTq^2 - 2zq + 2bR)d]}{aTq^2\omega(\beta_2)} \quad (17)$$

From (17) we can find d.

$$d = \frac{aTq^2[\bar{r}Z(\alpha_1) + \bar{d}\omega(\beta_2)]}{aTq^2[Z(\alpha_1) + \omega(\beta_2)] - 2Z(\alpha_1)zq + 2Z(\alpha_1)bR}$$

Substituting d to the left hand side of (17), it follws that

$$\mu_{G_q(\tilde{d},\tilde{r})}(z) = PQ = \frac{(1-\alpha)K_1(z)}{H_1(z)} \quad (18)$$

where

$H_1(z) = Z(\alpha_1)[aTq^2(Z(\alpha_1) + \omega(\beta_2)) - 2Z(\alpha_1)qz + 2Z(\alpha_1)bR]$ and

$K_1(z) =$

$aTq^2[\bar{r}Z(\alpha_1) + \bar{d}\omega(\beta_2)] - (\bar{d} - Z(\alpha_1))[aTq^2(Z(\alpha_1) + \omega(\beta_2)) - 2Z(\alpha_1)qz + 2Z(\alpha_1)bR]$

The existence of the region of z can be determined from

$P_2 \le \bar{d}$, $\bar{d} - Z(\alpha_1) \le P_3$, $\dfrac{bR}{q} < z \le \dfrac{aTq^2 + 2bR}{2q}$ (in (14)) , and

$0 < aTq^2 - 2zq + 2bR$ (in (13)) .

Let $S_1 = \dfrac{1}{2q(\bar{d} - Z(\alpha_1))}[aTq^2(\bar{d} - Z(\alpha_1) - \bar{r} - \omega(\beta_2)) + 2bR(\bar{d} - Z(\alpha_1))]$ and

$$S_2 = \frac{aTq^2(\bar{d} - \bar{r}) + 2bR\bar{d}}{2q\bar{d}} > 0 \quad (\because \quad 0 < r_j < d_j, \quad j = 1,2,...,n \Rightarrow \bar{r} < \bar{d})$$

(1^*) By $P_2 < \bar{d}$, we have $z \le S_2$ and $\frac{bR}{q} \le S_2 \le \frac{aTq^2 + 2bR}{2q}$

(2^*) By $\bar{d} - Z(\alpha_1) \le P_3$ we get

(2^*-1) if $\bar{d} - Z(\alpha_1) > 0$,then $S_1 \le z$ and satisfies $S_1 \le \frac{aTq^2 + 2bR}{2q}$,

(2^*-1-1) if $\bar{d} - Z(\alpha_1) - \bar{r} - \omega(\beta_2) > 0$, then $\frac{bR}{q} \le S_1$,

(2^*-1-2) if $\bar{d} - Z(\alpha_1) - \bar{r} - \omega(\beta_2) < 0$,then $\frac{bR}{q} \ge S_1$,

(2^*-2) If $\bar{d} - Z(\alpha_1) < 0$, then $z \le S_1$ and satisfies $\frac{aTq^2 + 2bR}{2q} \le S_1$

Then we have

(A-1) If $\bar{d} - Z(\alpha_1) > 0$, $\bar{d} - Z(\alpha_1) - \bar{r} - \omega(\beta_2) > 0$ and $S_1 < S_2$,
then $S_1 \le z \le S_2$ (19)

(A-2) If $\bar{d} - Z(\alpha_1) > 0$, $\bar{d} - Z(\alpha_1) - \bar{r} - \omega(\beta_2) < 0$, then

$$\frac{bR}{q} \le z \le S_2 \tag{20}$$

(A-3) If $\bar{d} - Z(\alpha_1) < 0$, then

$$\frac{bR}{q} \le z \le S_2 \tag{21}$$

〈 **Insert Figure 3 here** 〉

From figure 3, the second formula of (8) and the first formula of (16) we obtain

$$\frac{(1-\alpha)[\bar{d} + Z(\alpha_2) - d]}{Z(\alpha_2)} = \frac{(1-\alpha)[(aTq^2 - 2zq + 2bR)d - aTq^2(\bar{r} - \omega(\beta_1))]}{aTq^2\omega(\beta_1)} \tag{22}$$

From (22), we find

$$d = \frac{aTq^2[\bar{r}Z(\alpha_2) + \bar{d}\omega(\beta_1)]}{aTq^2[Z(\alpha_2) + \omega(\beta_1)] - 2Z(\alpha_2)zq + 2Z(\alpha_2)bR}$$

Substituting *d* to the left hand side of (22), we obtain

$$\mu_{G_q(\tilde{d},\tilde{r})}(z) = P'Q' = \frac{(1-\alpha)K_2(z)}{H_2(z)}, \tag{23}$$

where

$H_2(z) = Z(\alpha_2)[aTq^2(Z(\alpha_2) + \omega(\beta_1)) - 2Z(\alpha_2)qz + 2Z(\alpha_2)bR]$ and

$K_2(z) =$

$(\bar{d} + Z(\alpha_2))[aTq^2(Z(\alpha_2) + \omega(\beta_1)) - 2Z(\alpha_2)qz + 2Z(\alpha_2)bR] - aTq^2[\bar{r}Z(\alpha_2) + \bar{d}\omega(\beta_1)]$

The region of *z* can be determined by $\bar{d} \le P_2$, $P_1 \le \bar{d} + Z(\alpha_2)$ and $\frac{bR}{q} < z < \frac{aTq^2 + 2bR}{2q}$

Since $0 < aTq^2 - 2zq + 2bR$ (in (13)), then we have the following results

Let $S_3 = \frac{1}{2q[\bar{d} + Z(\alpha_2)]}[aTq^2(\bar{d} + Z(\alpha_2) - \bar{r} + \omega(\beta_1)) + 2bR(\bar{d} + Z(\alpha_2))]$

By $\bar{d} \le P_2$ we have $S_2 \le z$ and by $P_1 \le \bar{d} + Z(\alpha_2)$, we get $z \le S_3$.

It follows that $\frac{bR}{q} \le S_2 \le \frac{aTq^2 + 2bR}{2q}$

If $-\bar{r} + \omega(\beta_1) > 0$, then $\frac{aTq^2 + 2bR}{2q} \le S_3$

If $-\bar{r} + \omega(\beta_1) < 0$, then $\frac{aTq^2 + 2bR}{2q} \ge S_3$

Therefore we obtain

(B-1) if $-\bar{r} + \omega(\beta_1) > 0$ then

$$S_2 \le z \le \frac{aTq^2 + 2bR}{2q} \tag{24}$$

(B-2) if $S_2 < S_3$ and $-\bar{r} + \omega(\beta_1) < 0$ then

$$S_2 \le z \le S_3 \tag{25}$$

From (18)～(21) and (23)～(25), we have the following property.

Property2. For any $q > 0$,

(1^o) if $\bar{d} - Z(\alpha_1) > 0$, $\bar{d} - Z(\alpha_1) - \bar{r} - \omega(\beta_2) > 0$, $S_1 < S_2 < S_3$ and satisfies

(1^o-1) $-\bar{r} + \omega(\beta_1) > 0$, then

$$\mu_{G_q(\tilde{d},\tilde{r})}(z) = \begin{cases} \dfrac{(1-\alpha)K_1(z)}{H_1(z)} & , \quad S_1 \le z \le S_2 \\ \dfrac{(1-\alpha)K_2(z)}{H_2(z)} & , \quad S_2 \le z < \dfrac{aTq^2 + 2bR}{2q} \\ 0 & , \quad otherwise \end{cases} \tag{26}$$

(1^o-2) $-\bar{r} + \omega(\beta_1) < 0$, then

$$\mu_{G_q(\tilde{d},\tilde{r})}(z) = \begin{cases} \dfrac{(1-\alpha)K_1(z)}{H_1(z)} & , \quad S_1 \le z \le S_2 \\ \dfrac{(1-\alpha)K_2(z)}{H_2(z)} & , \quad S_2 \le z < S_3 \\ 0 & , \quad otherwise \end{cases} \tag{27}$$

(2^o) if $\bar{d} - Z(\alpha_1) > 0$, $\bar{d} - Z(\alpha_1) - \bar{r} - \omega(\beta_2) < 0$, $S_2 < S_3$ and satisfies

(2^o-1) $-\bar{r} + \omega(\beta_1) > 0$, then

$$\mu_{G_q(\tilde{d},\tilde{r})}(z)=\begin{cases}\dfrac{(1-\alpha)K_1(z)}{H_1(z)} & , \quad \dfrac{bR}{q}\le z\le S_2\\ \dfrac{(1-\alpha)K_2(z)}{H_2(z)} & , \quad S_2\le z<\dfrac{aTq^2+2bR}{2q}\\ 0 & , \quad otherwise\end{cases} \tag{28}$$

(2^o-2) $-\bar{r}+\omega(\beta_1)<0$, then

$$\mu_{G_q(\tilde{d},\tilde{r})}(z)=\begin{cases}\dfrac{(1-\alpha)K_1(z)}{H_1(z)} & , \quad \dfrac{bR}{q}< z\le S_2\\ \dfrac{(1-\alpha)K_2(z)}{H_2(z)} & , \quad S_2\le z<S_3\\ 0 & , \quad otherwise\end{cases} \tag{29}$$

(3^o) if $\bar{d}-Z(\alpha_1)<0$, $S_2<S_3$ and satisfies

(3^o-1) $-\bar{r}+\omega(\beta_1)>0$, then

$$\mu_{G_q(\tilde{d},\tilde{r})}(z)=\begin{cases}\dfrac{(1-\alpha)K_1(z)}{H_1(z)} & , \quad \dfrac{bR}{q}< z\le S_2\\ \dfrac{(1-\alpha)K_2(z)}{H_2(z)} & , \quad S_2\le z<\dfrac{aTq^2+2bR}{2q}\\ 0 & , \quad otherwise\end{cases} \tag{30}$$

(3^o-2) $-\bar{r}+\omega(\beta_1)<0$, then

$$\mu_{G_q(\tilde{d},\tilde{r})}(z)=\begin{cases}\dfrac{(1-\alpha)K_1(z)}{H_1(z)} & , \quad \dfrac{bR}{q}< z\le S_2\\ \dfrac{(1-\alpha)K_2(z)}{H_2(z)} & , \quad S_2\le z\le S_3\\ 0 & , \quad otherwise\end{cases} \tag{31}$$

3 The centroid of fuzzy total cost

Let $A_1=Z(\alpha_1)[aTq^2(Z(\alpha_1)+\omega(\beta_2))+2Z(\alpha_1)bR]$ and

$B_1=aTq^2[\bar{r}Z(\alpha_1)+\bar{d}\omega(\beta_2)]-(\bar{d}-Z(\alpha_1))[aTq^2(Z(\alpha_1)+\omega(\beta_2))+2Z(\alpha_1)bR]$

Then $H_1(z)=A_1-2Z(\alpha_1)^2qz$

$K_1(z)=B_1+2(\bar{d}-Z(\alpha_1))Z(\alpha_1)qz$

Define $T_1(a_1,a_2)\equiv\int_{a_1}^{a_2}\dfrac{K_1(z)}{H_1(z)}dz$, then

$$T_1(a_1,a_2)=\int_{a_1}^{a_2}[-\frac{\bar{d}-Z(\alpha_1)}{Z(\alpha_1)}+\frac{B_1+\dfrac{\bar{d}-Z(\alpha_1)}{Z(\alpha_1)}A_1}{A_1-2Z(\alpha_1)^2qz}]dz$$

$$=\frac{\bar{d}-Z(\alpha_1)}{Z(\alpha_1)}(a_1-a_2)+\frac{1}{2Z(\alpha_1)^2q}[B_1+$$

$$\frac{\bar{d}-Z(\alpha_1)}{Z(\alpha_1)}A_1][\ln|A_1-2Z(\alpha_1)^2a_1q|-\ln|A_1-2Z(\alpha_1)^2a_2q|] \quad (32)$$

Define $T_{12}(a_1,a_2)\equiv\int_{a_1}^{a_2} z\frac{K_1(z)}{H_1(z)}dz$, then

$$T_{12}(a_1,a_2)=\int_{a_1}^{a_2}[-\frac{\bar{d}-Z(\alpha_1)}{Z(\alpha_1)}z-\frac{1}{2Z(\alpha_1)^2q}(B_1+\frac{\bar{d}-Z(\alpha_1)}{Z(\alpha_1)}A_1)$$
$$+\frac{A_1}{2Z(\alpha_1)^2q}(B_1+\frac{\bar{d}-Z(\alpha_1)}{Z(\alpha_1)}A_1)\frac{1}{A_1-2Z(\alpha_1)^2qz}]dz$$
$$=\frac{\bar{d}-Z(\alpha_1)}{2Z(\alpha_1)}(a_1^2-a_2^2)+\frac{1}{2Z(\alpha_1)^2q}(B_1+\frac{\bar{d}-Z(\alpha_1)}{Z(\alpha_1)}A_1)(a_1-a_2)$$
$$+\frac{A_1}{4Z(\alpha_1)^4q^2}(B_1+\frac{\bar{d}-Z(\alpha_1)}{Z(\alpha_1)}A_1)[\ln|A_1-2Z(\alpha_1)^2a_1q|$$
$$-\ln|A_1-2Z(\alpha_2)^2a_2q|] \quad (33)$$

Let $A_2=Z(\alpha_2)[aTq^2(Z(\alpha_2)+\omega(\beta_1))+2Z(\alpha_2)bR]$ and

$$B_2=(\bar{d}+Z(\alpha_2))[aTq^2(Z(\alpha_2)+\omega(\beta_1))+2Z(\alpha_2)bR]-aTq^2(\bar{r}Z(\alpha_1)+\bar{d}\omega(\beta_2))$$

Then $H_2(z)=A_2-2Z(\alpha_2)^2qz$

$K_2(z)=B_2-2(\bar{d}+Z(\alpha_2))Z(\alpha_2)qz$

We will do the same arguments as (32) and (33).

Define $T_2(a_1,a_2)\equiv\int_{a_1}^{a_2}\frac{K_2(z)}{H_2(z)}dz$. Then

$$T_2(a_1,a_2)=\frac{\bar{d}+Z(\alpha_2)}{Z(\alpha_2)}(a_2-a_1)+\frac{1}{2Z(\alpha_2)^2q}(B_2-\frac{\bar{d}+Z(\alpha_2)}{Z(\alpha_2)}A_2)[\ln|A_2-2Z(\alpha_2)^2a_1q|$$
$$-\ln|A_2-2Z(\alpha_2)^2a_2q|] \quad (34)$$

$$T_{22}(a_1,a_2)=\int_{a_1}^{a_2}z\frac{K_2(z)}{H_2(z)}dz$$
$$=\frac{\bar{d}+Z(\alpha_2)}{2Z(\alpha_2)}(a_2^2-a_1^2)+\frac{1}{2Z(\alpha_2)^2q}(B_2-\frac{\bar{d}+Z(\alpha_2)}{Z(\alpha_2)}A_2)(a_1-a_2)$$
$$+\frac{A_2}{4Z(\alpha_2)^4q^2}(B_2-\frac{\bar{d}+Z(\alpha_2)}{Z(\alpha_2)}A_2)[\ln|A_2-2Z(\alpha_2)^2a_1q|$$
$$-\ln|A_2-2Z(\alpha_2)^2a_2q|] \quad (35)$$

Since $\lim_{a_1\to\frac{bR}{q}}T_j(a_1,a_2)=T_j(\frac{bR}{q},a_2)$ and $\lim_{a_1\to\frac{bR}{q}}T_{j2}(a_1,a_2)=T_{j_2}(\frac{bR}{q},a_2)$,

$j=1,2$, then letting $c=\frac{aTq^2+2bR}{2q}$, we also have

$$\lim_{a_2\to c}T_j(a_1,a_2)=T_j(a_1,c) \text{ and } \lim_{a_2\to c}T_{j_2}(a_1,a_2)=T_{j2}(a_1,a_2)$$

Using (32)～(35) we find the following centroid

The centroid of (26) is

$$c_{11}(q)=\frac{\int_{-\infty}^{\infty} z\mu_{G_q(\tilde{d},\tilde{r})}(z)dz}{\int_{-\infty}^{\infty}\mu_{G_q(\tilde{d},\tilde{r})}(z)dz}=\frac{(1-\alpha)[T_{12}(S_1,S_2)+T_{22}(S_2,\frac{(aTq^2+2bR)}{2q})}{(1-\alpha)[T_1(S_1,S_2)+T_2(S_2,\frac{(aTq^2+2bR)}{2q})}$$

$$=\frac{T_{12}(S_1,S_2)+T_{22}(S_2,(\frac{aTq^2+2bR}{2q}))}{T_1(S_1,S_2)+T_2(S_2,(\frac{aTq^2+2bR}{2q}))}$$

$c_{11}(q)$ represents the estimate of the total cost of the production quantity q per cycle when

$\bar{d}-Z(\alpha_1)>0$, $\bar{d}-Z(\alpha_1)-\bar{r}-\omega(\beta_2)>0$, $S_1<S_2<S_3$ and $-r+\omega(\beta_1)>0$

We can apply the same arguments to other cases.

Property 3 Centroid of $\mu_{G_q(\tilde{d},\tilde{r})}(z)$

(1^o) If $\bar{d}-Z(\alpha_1)>0$, $\bar{d}-Z(\alpha_1)-\bar{r}-\omega(\beta_2)>0$, $S_1<S_2<S_3$ and satisfies

(1^o-1) $-\bar{r}+\omega(\beta_1)>0$, then

$$c_{11}(q)=\frac{T_{12}(S_1,S_2)+T_{22}(S_2,(\frac{aTq^2+2bR}{2q}))}{T_1(S_1,S_2)+T_2(S_2,(\frac{aTq^2+2bR}{2q}))} \tag{36}$$

(1^o-2) $-\bar{r}+\omega(\beta_1)<0$, then

$$c_{12}(q)=\frac{T_{12}(S_1,S_2)+T_{22}(S_2,S_3)}{T_1(S_1,S_2)+T_2(S_2,S_3)} \tag{37}$$

(2^o) If $\bar{d}-Z(\alpha_1)>0$, $\bar{d}-Z(\alpha_1)-\bar{r}-\omega(\beta_2)<0$, $S_2<S_3$ and satisfies

(2^o-1) $-\bar{r}+\omega(\beta_1)>0$, then

$$c_{21}(q)=\frac{T_{12}(\frac{bR}{q},S_2)+T_{22}(S_2,(\frac{aTq^2+2bR}{2q}))}{T_1(\frac{bR}{q},S_2)+T_2(S_2,(\frac{aTq^2+2bR}{2q}))} \tag{38}$$

(2^o-2) $-\bar{r}+\omega(\beta_1)<0$, then

$$c_{22}(q)=\frac{T_{12}(\frac{bR}{q},S_2)+T_{22}(S_2,S_3)}{T_1(\frac{bR}{q},S_2)+T_2(S_2,S_3)} \tag{39}$$

(3^o) If $\bar{d}-Z(\alpha_1)<0$, $S_2<S_3$ and satisfies

(3^o-1) $-\bar{r}+\omega(\beta_1)>0$, then

$$c_{31}(q)=\frac{T_{12}(\frac{bR}{q},S_2)+T_{22}(S_2,(\frac{aTq^2+2bR}{2q}))}{T_1(\frac{bR}{q},S_2)+T_2(S_2,(\frac{aTq^2+2bR}{2q}))} \quad (40)$$

(3^o-2) $-\bar{r}+\omega(\beta_1)<0$, then

$$c_{32}(q)=\frac{T_{12}(\frac{bR}{q},S_2)+T_{22}(S_2,S_3)}{T_1(\frac{bR}{q},S_2)+T_2(S_2,S_3)} \quad (41)$$

$c_{ij}(q)$, $i=1,2,3$, $j=1,2$ are the total cost in the fuzzy sense under different cases.

4 Example

Give $a=10$, $b=500$, $T=40$ and $R=80$. Suppose we get $n=5$ data points. The demand quantities per day are $r_1=2.4, r_2=3, r_3=2.8, r_4=3.2$ and $r_5=2.6$. The production quantities per day are $d_1=8.6$, $d_2=9$, $d_3=9.4$, $d_4=9.3$ and $d_5=8.7$. We find $\bar{r}=\frac{1}{5}\sum_{j=1}^{5}r_j=2.8$ and $\bar{d}=\frac{1}{5}\sum_{j=1}^{5}d_j=9$, $\mu_1^2=\frac{1}{4}\sum_{j=1}^{5}(d_j-\bar{d})^2=0.1$ and $\mu_2^2=\frac{1}{4}\sum_{j=1}^{5}(r_j-\bar{r})^2=0.125$.

From property 1, the optimal solution is $F(q_0)=4695.15$ when $q_0=17.04$

Let $\alpha=0.05$, $\alpha_1=0.06$, $\alpha_2=0.04$. Then $\alpha_1+\alpha_2=2\alpha$, $t_4(\alpha_1)=2.647$, $t_4(\alpha_2)=3.1$,

Let $\beta_1=0.08$ and $\beta_2=0.02$. Then $\beta_1+\beta_2=2\alpha$, $t_4(\beta_1)=2.390$, $t_4(\beta_2)=3.747$

$Z(\alpha_1)=t_4(\alpha_1)\frac{\mu_1}{\sqrt{5}}=0.3743$, $Z(\alpha_2)=t_4(\alpha_2)\frac{\mu_1}{\sqrt{5}}=0.4384$

$\omega(\beta_1)=t_4(\beta_1)\frac{\mu_2}{\sqrt{5}}=0.3779$, $\omega(\beta_2)=t_4(\beta_2)\frac{\mu_2}{\sqrt{5}}=0.5925$

Then we have level 0.95 fuzzy numbers

$\tilde{d}=(9-0.3743\ ,\ 9\ ,\ 9+0.4384\ ;\ 0.95)=(8.6257\ ,\ 9\ ,\ 9.4384\ ;\ 0.95)$

$\tilde{r}=(2.8-0.3779\ ,\ 2.8\ ,\ 2.8+0.5928\ ;\ 0.95)=(2.4221\ ,\ 2.8\ ,\ 3.3928\ ;\ 0.95)$.

Using computer, we get the following results (by property 2, 3)

q	c
15.9	4815.195
16.0	4813.934
16.1	4812.868
16.2	4811.994
16.3	4811.308
16.4	4810.807
16.5	4810.487
16.6	4810.345
16.7	4810.378

16.8	4810.583
16.9	4810.957
17.0	4811.496
17.1	4812.198
17.2	4813.061
17.3	4814.080
17.4	4815.254
17.5	4816.580
17.6	4818.055
17.7	4819.677
17.8	4821.442
17.9	4823.351

The minimum total cost in the fuzzy sense occurs about $q = 16.6$ while it occurs about $q = 17.04$ in crisp case. $\frac{16.6 - 17.04}{17.04}$ × 100% = -2.5%

$$\frac{4810.345 - 4695.15}{4695.15} \times 100\% = 2.4\%$$

5 Concluding remarks

This liturature uses the statistical data in the past and fuzzy concept to treat the inventory problem. It seems more practical and meaningful in applications.

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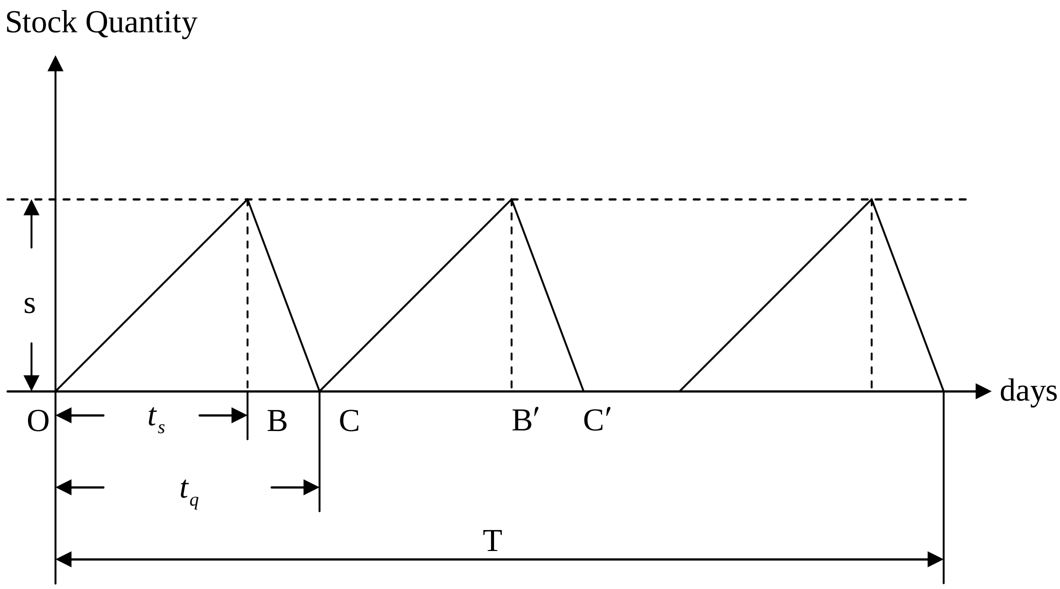

Figure 1 Production Inventory Model

[A]

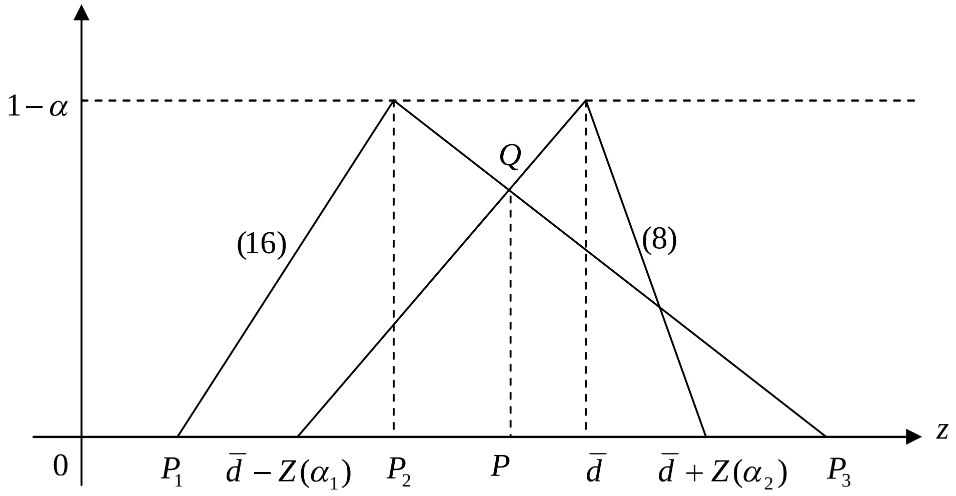

Figure 2 Graph of Case 1

[B] $\bar{d} \le P_2$ and $P_1 \le \bar{d} + Z(\alpha_2)$

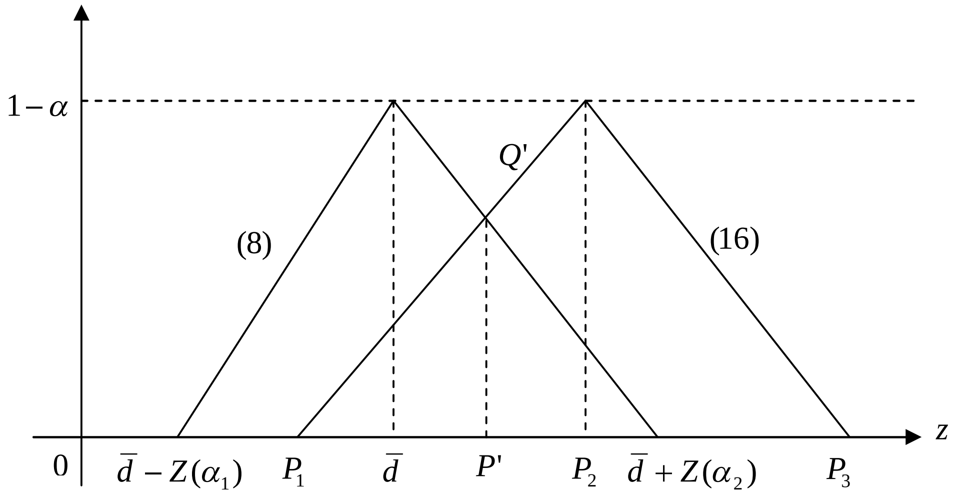

Figure 3 Graph of Case 2