行政院國家科學委員會補助專題研究計畫成果報告

※※※※※※※※※※※※※※※※※※※※※※※※※
※ ※
※ 多段連續－2線的循序分派 ※
※ Sequential assignments for many consecutive-2 lines ※
※ ※
※※※※※※※※※※※※※※※※※※※※※※※※※

計畫類別：個別型計畫

計畫編號：NSC 89－2115－M－034－002－

執行期間：89 年 8 月 1 日 至 90 年 7 月 31 日

計畫主持人：何志昌

本成果報告包括以下應繳交之附件：

□赴國外出差或研習心得報告一份

□赴大陸地區出差或研習心得報告一份

□出席國際學術會議心得報告及發表之論文各一份

□國際合作研究計畫國外研究報告書一份

執行單位：中國文化大學 應用數學系

中 華 民 國 90 年 10 月 29 日

行政院國家科學委員會專題研究計畫成果報告

多段連續－2線的循序分派

Sequential assignments for many consecutive-2 lines

計畫編號：NSC 89－2115－M－034－002－

執行期限：89年8月1日至90年7月31日

主持人：何志昌　執行單位：中國文化大學 應用數學系

一、中文摘要

考慮一個包含了 t 條線及 n 個位置的系統。其中若在某一條線中，有某兩個相鄰的元件都故障，則整個系統就算是故障。假設元件 i 的故障率為 q(i)=1-p(i)；每一個元件一旦被填入系統，立刻可被測得它是否故障。如何將故障率都不一樣的 n 個元件填入系統中，使得系統的故障率為最小，便成為一個重要的問題。

填入的順序可以有不同的限制。如果我們進一步假設填入的順序為：先填入第一條線的第一個位置，再填入第二個位置・・・，一直到第一條線都填完，再填入第二條線的第一個位置、第二個位置・・・，依此類推。符合此條件限制的填入方法，稱為一個循序分派(或循序配置)。

如果有一個最佳的循序分派只和各個 p(i)的大小順序有關，和各個 p(i)的值無關，我們稱此分派為守恒的。目前已知的結果是：如果只有一條線時，守恒的循序分派是存在的。在本計畫中，我們證明在一般情形、t 條線 (m>1) 時，守恒的循序分派是不存在的。

關鍵詞：循序分派、循序配置、連續-貳線

Abstract

Consider a system of t lines where the system is failed if and only if there exist two consecutive failed nodes on some line. The problem is to assign items with different reliabilities p_j to the nodes to maximize the system reliability.

In a sequential assignment, the t lines are ordered and the nodes in a line are ordered naturally. Thus the n nodes are ordered linearly to receive assignment and the state of an item (working or failed) becomes known once it is assigned to the node.

An optimal assignment is invariant if it depends only on the ranks of p_j, but not their values. Invariant sequential assignment is known to exist for the 1-line system. In this project, we show that invariant assignment does not exist in general for the t-line system if $t>1$.

Keywords: Sequential Assignment,
Consecutive-2 Line

二、計畫緣由與目的

Let $S\{k_1, \ldots, k_t\}$ denote a system of m lines with lengths $k_1, \ldots, k_t$. Let N be a set of $n=\sum_{i=1}^{t} k_i$ items $I_1, \ldots, I_n$, where I_i has probability p_i of working and probability $q_i = 1 - p_i$ of failure. Without loss of generality, we assume $p_1 \le p_2 \le \cdots \le p_n$. The n items are to be assigned to the n nodes in S, which fails if and only if there exist two failed nodes adjacent in a line of S. According to the order of nodes receiving assignments, Hwang [3] suggests three different assignment problems. **(i) Sequential assignment.** The t lines are ordered and the nodes in a line are ordered naturally. Thus the n nodes are ordered linearly to receive assignment. Let $S(k_1, \ldots, k_t)$ denote a system to receive a sequential assignment and $R^A(k_1, \ldots, k_t)$ its reliability (probability of system working under assignment A). ***(ii)* Permutational assignment.** To find a permutation of the lines followed by a sequential assignment under that permutation. Let $S\{k_1, \ldots, k_t\}$ denote a system to receive a permutational assignment. **(iii) Dynamic assignment.** The n nodes can receive assignment in any order. Let $S[k_1, \ldots, k_t]$ denote a system to receive a dynamic assignment.

A sequential (permutation, dynamic) assignment is optimal if it maximizes the reliability over all assignments in the respective class. Let $S^*(\ )$ ($S^*\{\ \}$, $S^*[\ ]$) denote an optimal assignment, and $R^*(\ )$ ($R^*\{\ \}$, $R^*[\ ]$) its reliability. An optimal assignment is called *invariant* if it depends only on the ranking of $p_1, \ldots, p_n$, but not their values. For $S\{n\}$ consisting of a single line, sequential assignment and permutation assignment are the same. Derman, Lieberman and Ross [1] gave an invariant assignment for $S(n)$=$S\{n\}$. Ho and Hwang [2] proved that no invariant assignment exists for $S[n]$, but they reduced the number of candidates for optimal dynamic assignment to $\lfloor n/2 \rfloor$.

For sequential assignment on many lines, Hwang [3] proved

Theorem. There exists an optimal sequential assignment for $S(n)$ in which a failed component is followed by the most reliable item among the unassigned.

Let S_i^* denote an optimal assignment among those which assign I_i first, and let R_i^* denote the reliability of S_i^*. Derman, Lieberman and Ross [1] proved that $S^*(n)=S_1^*(n)$. Hwang [3] generalized this result and showed

Theorem. $S^*(n)=S_i^*(n)$ in the invariant sense if and only if $1 \le i \le \lfloor n/2 \rfloor +1$.

For 2-line system, he also proved

Theorem.
$R^*\{2,k\}=R_2^*(k,2)=R_3^*(k,2)=\cdots=R_{\lfloor n/2 \rfloor}^*(k,2)$ and no other permutational assignment is invariant.

In this project we study invariant sequential assignments for general system $S(k_1, \ldots, k_l)$.

三、結果與討論

For the sequential assignments on the system $S(k_1,k_2)$ with $k_2 \le 5$, we prove

Theorem 1.

(i) If $k \ge 1$, then $R^*(k,1)=R_i^*(k,1)$ for $2 \le i \le \lfloor k/2 \rfloor +2$.

(ii) If $k \ge 2$, then $R^*(k,2)=R_i^*(k,2)$ for $2 \le i \le \lfloor k/2 \rfloor +1$.

(iii) If $k \ge 2$, then $R^*(k,3)=R_i^*(k,3)$ for $3 \le i \le \lfloor k/2 \rfloor +2$.

(iv) If $k \ge 4$, then $R^*(k,4)=R_i^*(k,4)$ for $3 \le i \le \lfloor k/2 \rfloor +1$.

(v) If $k \ge 4$, then $R^*(k,5)=R_i^*(k,5)$ for $4 \le i \le \lfloor k/2 \rfloor +2$.

Note that Theorem 1 implies that

Corollary 2. Invariant sequential assignments exist for the system $S(k_1,k_2)$, if $k_1 \ge 1$ and $k_2 \le 5$.

Hwang [3] pointed out that no invariant assignment exists for the system S(2,6), since $R_1^*(2,6) \le R_2^*(2,6) \le R_3^*(2,6) \le R_4^*(2,6) \le R_5^*(2,6)$, $R_6^*(2,6) \ge R_7^*(2,6) \ge R_8^*(2,6)$, and $R_5^*(2,6)-R_6^*(2,6)=$
$(p_6-p_5)p_8(-q_1p_2p_3p_4q_7+p_1q_2q_3q_4p_7)$,
where the sign of $R_5^*(2,6)-R_6^*(2,6)$ does not only depend on the ranking of p_i's (assuming $p_1 \le p_2 \le \cdots \le p_8$), but also depends on their values. We say such two reliabilities are *incomparable*. In general, for $S(2,k)$ system we show

Theorem 3.

(i) $R_i^*(2,k) \le R_{i+1}^*(2,k)$, if $k \ge 4$ and $1 \le i \le \lceil k/2 \rceil +1$.

(ii) $R_i^*(2,k) \ge R_{i+1}^*(2,k)$, if $k \ge 4$ and $i=k$ or $k+1$.

(iii) $R_i^*(2,k)$ and $R_{i+1}^*(2,k)$ are incomparable, if $k \ge 6$ and $\lceil k/2 \rceil +2 \le i \le k-1$.

Note that from Theorem 3, there does not exist an invariant sequential assignment for any $S(2,k)$ system with $k \ge 6$. We also note that the nonexistence of an invariant sequential assignment for an $S(i,k)$ system implies the nonexistence of an invariant sequential assignment for an $S(i+1,k)$ system. Combining these and Corollary 2, we have

Theorem 4. For an $S(k_1,k_2)$ system, no invariant sequential assignments exist if and only if $k_1 \ge 2$ and $k_2 \ge 6$.

Finally, since a t-line system for $t \geq 3$ contains a 2-line system as a subsystem, and since the nonexistence of invariant assignment to a subsystem implies the same for the system, we conclude that no invariant assignments exist for $S(k_1, \ldots, k_t)$ in general for $t \geq 3$.

四、計畫成果自評

The goal of this project is to prove Theorems 1 and 4. Both proofs are found. Not only we study the invariant sequential assignments for 2-line system, but also we have the results for general t-line system, $t \geq 3$.

五、參考文獻

1. **C. Derman, G. J. Lieberman, and S. M. Ross**, On the consecutive-k-of-n:F system, *IEEE Trans. Rel.* **31** (1982),57-63.

2. **C-C. Ho and F. K. Hwang**, Dynamic assignments on consecutive-2 system, preprint, 1999.

3. **F. K. Hwang**, personal communication.